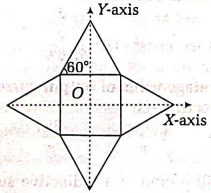
Solved Paper 2015
CG PET
Engineering Entrance Exam

Physics

1. If 10g of Uranium-235 is completely destroyed in a reactor, then energy released is
(a) 9×10^{14} J (b) 9×10^{15} J
(c) 9×10^{16} J (d) 9×10^{18} J

2. In which of the following process, convection does not take place primarilly?
(a) Sea and land breeze
(b) Boiling of water
(c) Warming of glass bulb due to filament
(d) Heating air around furnace

3. Moment of inertia of a uniform symmetric plate as shown in figure about Y-axis is I. Moment of inertia of this plate about an axis passing through the centre of plate O and perpendicular to the plane of plate is



(a) $2I$ (b) $\frac{I}{2}$
(c) $\frac{I}{4}$ (d) I

4. The horizontal component of earth's magnetic field at a place is 0.4×10^4 T. If the angle of dip is 45° the value of total intensity of earth's magnetic field is
(a) 0.5×10^{-4} T (b) 0.4×10^{-4} T
(c) 0.5×10^{-6} T (d) 0.4×10^{-6} T

5. Three identical bodies of equal mass M each are moving along a circle of radius R under the action of their mutual gravitational interaction. The speed of each body is
(a) $\sqrt{\frac{GM}{\sqrt{3}R}}$ (b) $\sqrt{\frac{GM}{3R}}$
(c) $\sqrt{\frac{GM}{R}}$ (d) $\sqrt{\frac{GM}{\sqrt{2}R}}$

6. If an observer moves towards stationary source, then the apparent frequency is given by
(a) $f' = f\left(\frac{v + v_0}{v}\right)$ (b) $f' = f\left(\frac{v - v_0}{v}\right)$
(c) $f' = f\left(\frac{v}{v + v_0}\right)$ (d) $f' = f\left(\frac{v}{v - v_0}\right)$

7. Two coils have mutual inductance of 1.5 H. If current in primary coil is suddenly raised to 5 A in one milli second, the induced emf in the secondary coil is
(a) 75 V (b) 750 V
(c) 7500 V (d) 75000 V

8. Water rises to a height of 10 cm in a certain capillary tube. In the same tube, the level of mercury is depressed by 3.42 cm. The ratio of surface tensions of water and mercury is (specific gravity of mercury is 13.6, the angle of contact for water is zero and that for mercury is 135°)
(a) 1 : 1 (b) 1 : 13.6
(c) 1 : 6.57 (d) 1 : 0.342

9. The distance between H^+ and Cl^- ions in HCl molecules is 1.28Å. The potential due to this dipole at a distance of 12Å on the axis of dipole is
(a) 0.13 V (b) 1.3 V
(c) 13 V (d) 130 V

10. The formula for magnetic field of a certain component is given by $B = \frac{\mu_0 Ni}{2\pi r}$,

where, N = total number of turns
i = current through it
r = radius.

The component is
(a) ring (b) solenoid
(c) toroid (d) None of these

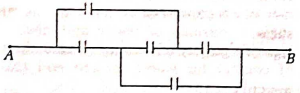
11. The temperature dependance of resistance is represented by which one of the following?
(a) $R(T) = R(T_0)[1 + \alpha(T_0 - T)]$
(b) $R(T) = R(T_0)[1 + (\alpha T_0 - T)]$
(c) $R(T) = R(T)[1 + \alpha(T - T_0)]$
(d) $R(T) = R(T_0)[1 + \alpha(T - T_0)]$

12. A convex lens of focal length 0.12 m produces an image, which is three times as long as the object. The distance between the object and the lens for a real image
(a) 0.16 m (b) – 0.16 m
(c) 1.6 m (d) – 1.6 m

13. An object is moving in a circle at constant speed v. The magnitude of the rate of change of momentum of the object is
(a) zero (b) proportional to v^2
(c) proportional to v^3 (d) proportional to v

14. A current passes through a wire of unequal cross-sectional area. The one that does not depend on cross-section among the following is
(a) free electron density (b) current density
(c) drift speed (d) None of these

15. Five equal capacitors each with capacitance C are connected as shown in figure. Then, the equivalent capacitance between A and B is



(a) $5C$ (b) $\frac{C}{5}$ (c) $3C$ (d) C

16. Angular momentum of an electron in an excited hydrogen atom having energy – 3.4 eV is
(a) 1.7×10^{-34} Js (b) 2.1×10^{-34} Js
(c) 3.9×10^{-34} Js (d) 4.1×10^{-35} Js

17. Two infinite plane parallel sheets separated by a distance d have equal and opposite uniform charge density σ. Electric field at a point between the sheet is
(a) zero
(b) $\frac{\sigma}{\varepsilon_0}$
(c) $\frac{\sigma}{2\varepsilon_0}$
(d) depends upon the location of the point

18. The time period of a freely suspended magnet is 4 s. If it is broken in length into two equal parts and one part is suspended in the same way, then the time period will be
(a) 4 s (b) 2 s
(c) 0.5 s (d) 0.25 s

19. Calculate the stress for one litre of a perfect gas at the pressure of 72 cm of Hg, when it is compressed isothermally to a volume of 900 cc.
(Take, $g = 10\ m/s^2$)
(a) $9.88 \times 10^3\ N/m^2$ (b) $10.88 \times 10^3\ N/m^2$
(c) $10.88 \times 10^3\ N/m^2$ (d) $2 \times 10^3\ N/m^2$

20. A spring of force constant k is cut into two pieces such that one piece is double the length of other. Then, the long piece will have a force constant of
(a) $\frac{2}{3}k$ (b) $\frac{3}{2}k$
(c) $3k$ (d) $6k$

21. The average value of output current is a half wave rectifier is
(a) zero (b) $\frac{I_0}{\pi}$ (c) $\frac{2I_0}{\pi}$ (d) $\frac{I_0}{\sqrt{2}}$

22. Half-life period of a radiactive substance is 10 min, then amount of substance decayed in 40 min will be
(a) 25%
(b) 50%
(c) 75%
(d) None of the above

23. Two charge particle of mass 4 m and m and having charges $+q$ and $+3q$ are placed in uniform electric field and allowed to move for 2 s. The ratio of KE acquired by them is
(a) 1 : 6 (b) 1 : 36 (c) 1 : 16 (d) 1 : 4

24. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each container is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are to be found Δp and 1.5 Δp respectively, then relation for masses will be
(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_a = 4m_B$

25. If one mole of a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of γ for the mixture is
(a) 1.40 (b) 1.50
(c) 1.53 (d) 3.07

26. Which of the following relation is correct?
(a) $\frac{C_p}{C_V} = R$ (b) $C_p + C_V = R$
(c) $C_p - C_V = R$ (d) None of these

27. A solid sphere of mass 2 kg rolls on a smooth horizontal surface at 10 m/s and then rolls up a smooth 30° incline. The maximum height reached by the sphere is ($g = 9.8\ m/s^2$)
(a) 10 m (b) 4.9 m (c) 14.2 m (d) 7.1 m

28. Galvanometer of coil resistance 1000 Ω gives full scale deflection for a current of 5 mA. What resistance and how it has to be connected to convert the galvanometer to a voltmeter to measure 50 V?
(a) 9000 Ω in series (b) 9000 Ω in parallel
(c) 9 Ω in series (d) 900 Ω in parallel

29. How much deep inside the earth (radius R) should a man go, so that his weight becomes one-fourth of that on the earth's surface?
(a) $\frac{R}{2}$ (b) $\frac{3R}{4}$ (c) $\frac{R}{4}$ (d) $\frac{R}{3}$

30. A circular coil of 300 turns and diameter 14 cm carries a current of 15 A. The magnitude of magnetic moment associated with this coil is nearly equal to
(a) 700 JT^{-1} (b) 7000 JT^{-1}
(c) 7 JT^{-1} (d) 70 JT^{-1}

31. Work function of a substance is 3.3 eV. Its threshold frequency is
(a) 4×10^{14} Hz (b) 8×10^{14} Hz
(c) 16×10^{20} Hz (d) 8×10^{20} Hz

32. In the reaction,

$$S_{nth} = u + \frac{a}{2}(2n - 1)$$

where, the letters have their usual meanings. The dimensions of S_{nth} are
(a) $[M^0 L^1 T^{-1}]$ (b) $[M^0 L^1 T^0]$
(c) $[M^1 L^0 T^1]$ (d) $[M^1 L^{-1} T^{-1}]$

33. Four projectiles are thrown with the same initial speed making angles 27°, 36°, 43°, 51°, with the horizontal. The range of projectiles will be largest for the projectile fired at angle
(a) 27° (b) 36°
(c) 43° (d) 51°

34. A transformer has 200 turns in primary and 150 turns in secondary. If the operating voltage at the load connected to the secondary is measured to be 300V, the voltage supplied at the primary is
(a) 230 V (b) 400 V
(c) 480 V (d) 660 V

35. A wire of resistance 5 Ω is drawn out so that its length is increased by twice its original length, its new resistance is
(a) 45 Ω (b) 54 Ω
(c) 9 Ω (d) 5 Ω

36. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Two objects each of mass m are attached gently on the opposite ends of the diameter of the ring. The angular velocity of the ring now will be
(a) ω
(b) $\frac{M + 2m}{M}\omega$
(c) $\frac{\omega M}{m}$
(d) $\frac{\omega M}{M + 2m}$

37. The magnifying power of a telescope is m'. If the focal length of the eye piece is doubled, then its magnifying power will become
(a) $\sqrt{2}\,m$ (b) $3m$ (c) $2m$ (d) $\frac{m}{2}$

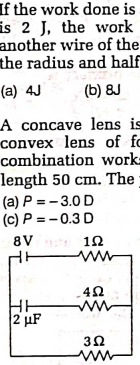
38. How will the interference pattern and fringe width in Young's double slit experiment get affected when the distance between the slits S_1 and S_2 reduced?
(a) Fringe width increases
(b) Fringe width decreases
(c) Fringe width remains same
(d) None of the above

39. On a heater coil it is written that, 250 V, 500 W. What is the resistance of this coil?
(a) 62.5 Ω (b) 100 Ω (c) 200 Ω (d) 125 Ω

40. When a dielectric slab is introduced between the plates of a capacitor connected to a battery, then
(a) charge on capacitor increases
(b) potential difference across the capacitor increases
(c) energy stored increases
(d) capacity remains the same

41. If the work done is stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of the same material but double the radius and half the length by 1 mm is
(a) 4J (b) 8J (c) 16J (d) $\frac{1}{2}$ J

42. A concave lens is kept in contact with a convex lens of focal length 20 cm. The combination works as convex lens of focal length 50 cm. The power of concave lens is
(a) $P = -3.0$ D (b) $P = +3.0$ D
(c) $P = -0.3$ D (d) $P = +0.3$ D

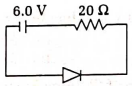
43.


In steady state, the charge on the 2 μF capacitor is equal to
(a) 8 μC (b) 12 μC (c) 6 μC (d) 0 μC

44. The critical angle for glass water interface $\left(\text{if } {}_a\mu_g = \frac{3}{2}, {}_a\mu_w = \frac{4}{3}\right)$
(a) $\sin^{-1}\left(\frac{8}{9}\right)$ (b) $\sin^{-1}\left(\frac{9}{8}\right)$
(c) $\sin^{-1}\left(\frac{3}{2}\right)$ (d) $\sin^{-1}\left(\frac{4}{3}\right)$

45. If wavelength of light used in Young's double slit experiment is 5000 Å, then phase difference between the waves reaching third bright fringe and central fringe will be
(a) zero (b) 2π
(c) 3π (d) 6π

46. A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be
(a) 225 (b) 450 (c) 900 (d) 1800

47. What is the current through the circuit and the pontential difference across the diode shown in figure. The drift current for the diode is 30 μA.


(a) 30 μ A , 5.99 V (b) 30 μ A, 5 V
(c) 20 μA , 6 V (d) 20 μA , 5.99 V

48. A particle of mass m is executing oscillation about the origin on x-axis. Its potential energy is $U(x) = K|x|^3$, where K is a positive constant. If the amplitude of oscillation is a, then its time period T is proportional to
(a) $\frac{1}{\sqrt{a}}$ (b) independent of a
(c) $\sqrt{a}$ (d) $a^{3/2}$

49. A man who wears glasses of power 3 D must holds a newspaper at least 25 cm away to see the print clearly. How far away would the newspaper have to be if he took off the glasses and still wanted clear vision?
(a) 10 cm (b) 25 cm
(c) 1 m (d) −1 m

50. The force per unit length between two parallel current carrying straight conductors separated by $2d$ is given by the formula
(a) $\frac{\mu_0}{4\pi}\frac{i_1 i_2}{d}$
(b) $\frac{\mu_0}{4\pi}\frac{i_1 i_2}{2d}$
(c) $\frac{\mu_0}{\pi}\frac{i_1 i_2}{2d}$
(d) None of the above

Chemistry

1. $A \xrightarrow{HOH/H^+}$ glucose + fructose
$B \xrightarrow{HOH/H^+}$ glucose + glucose
$C \xrightarrow{HOH/H^+}$ glucose + galactose
The disaccharides A, B and C respectively are
(a) lactose, sucrose and maltose
(b) sucrose, maltose and lactose
(c) sucrose, lactose and maltose
(d) maltose, sucrose and lactose

2. n/p ratio during positron decay
(a) increases (b) decreases
(c) remains constant (d) All of these

3. In the sequence of reaction,
$$A \xrightarrow{HOH/OH^-} R-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow{HOH/H^+} R-COOH$$
A starting compound A is
(a) RCN (b) RNC
(c) RCHO (d) R_2CO

4. Number of carbon atoms in kerosene is
(a) C_{17-20} (b) C_{12-16}
(c) C_{20-25} (d) C_{25-30}

5. A sample of gasoline contain 81% octane and 19% n-heptane. Its octane number will be
(a) 19 (b) 81 (c) 100 (d) 62

6. Which of the following polymers can be used for lubrication and as an insulator?
(a) SBR (b) PVC
(c) PTFE (d) PAN

7. What is the free energy change ΔG when 10 moles of water at 100° C and 1 atm pressure is converted into steam at 100° C and 1 atm pressure?
(a) 540 cal (b) − 9800 cal
(c) 9800 cal (d) 0 cal

8. Vapour pressure of a solution at 100° C having 3.42 g of cane sugar in 180 g water is
(a) 759.2 mm (b) 760 mm
(c) 740 mm (d) 748.5 mm

9. Among the following groupings which represent the collection of isoelectronic species.
(a) NO^+, C_2^{2-}, C_2^-, CO (b) N_2, C_2^{2-}, CO, NO
(c) CO, NO^+, CN^-, C_2^{2-} (d) NO, CN^-, N_2, O_2^-

10. The IUPAC name of


is
(a) 3-methyl cyclohexene
(b) 1-methyl cyclohex-2-ene
(c) 6-methyl cyclohexene
(d) 1-methyl cyclohex-5-ene

11. The correct representation of a complex ion is
(a) $[Co(H_2O)(NH_3)_4Cl]^{2+}$ (b) $[CoCl(H_2O)(NH_3)_4]^{2+}$
(c) $[Co(NH_3)_4Cl(H_2O)]^{2+}$ (d) $[Co(NH_3)_4(H_2O)Cl]^{2+}$

12. Chlorine is least reactive in
(a) methyl chloride (b) ethyl chloride
(c) vinyl chloride (d) allyl chloride

13. Which element has maximum first ionisation potential?
(a) Cs (b) F (c) Na (d) He

14. Among the following which one is condensation polymer?
(a) Orlon (b) Melamine
(c) Teflon (d) PMMA

15. In which of the following $\Delta E = \Delta H$?
(a) $N_2O_4(g) \rightleftharpoons 2NO_2(g)$
(b) $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$
(c) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$
(d) $H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(l)$

16. Tetrabromoethane on heating with Zn gives
(a) ethyl bromide (b) ethane
(c) ethyne (d) ethene

17. Calcination is used in metallurgy for removal of
(a) water and sulphide
(b) water and CO_2
(c) CO_2 and H_2S
(d) H_2O and H_2S

18. The empirical formula of a compound is CH_2. One mole of this compound has a mass 42g. Its molecular formula is
(a) C_3H_6 (b) C_2H_6 (c) CH_2 (d) C_2H_2

19. What is the value of x on the $[Ni(CN)_4]^x$ complex ion?
(a) +2 (b) − 2 (c) 0 (zero) (d) + 4

20. 75% of a first order reaction was completed in 32 min. When was 50% of the reaction completed?
(a) In 16 min (b) In 24 min
(c) In 8 min (d) In 4 min

21. The activation energy of a reaction is zero. The rate constant of this reaction
(a) increases with an increase of temperature
(b) decreases with an increase of temperature
(c) decreases with decrease of temperature
(d) is independent of temperature

22. Which of the following is a highly corrosive salt?
(a) $FeCl_2$ (b) $PbCl_2$
(c) Hg_2Cl_2 (d) $HgCl_2$

23. If NaCl is doped with 10^{-3} mol% of $SrCl_2$, the number of cation vacancies will be
(a) 6.023×10^{18} (b) 1×10^{-3}
(c) 6×10^{12} (d) 6.023×10^{23}

24. Which of the following does not exist?
I. $HO_3S—S—SO_3H$
II. HO—Te—OH
III. $HS—S_2—SH$
IV. HS—Po—OH
(a) Only II (b) Only III
(c) II and IV (d) I, III and IV

25. The velocity of a moving electron having 4.55×10^{-25} J of kinetic energy is
(a) 10^3 m s^{-1} (b) 10^5 cm s^{-1}
(c) 10^2 m s^{-1} (d) 10^4 cm s^{-1}

26. Which of the following notations shows the product incorrectly?
(a) $^{10}_{5}B(\alpha, n)\,^{13}_{7}N$
(b) $^{242}_{96}Cm(\alpha, 2n)\,^{243}_{97}Bk$
(c) $^{14}_{7}N(n, p)\,^{14}_{6}C$
(d) $^{28}_{14}Si(d, y)\,^{30}_{15}P$

27. In a reversible reaction a catalyst will affect the rate of
(a) forward reaction
(b) forward and reverse reaction
(c) reverse reaction
(d) Neither (a) nor (b)

28. Acidic dichromate ion reacts with hydrogen peroxide to give deep blue colour. This is due to the formation of
(a) $CrO(O)_2$ (b) CrO_5
(c) Both (a) and (b) (d) None of (a) and (b)

29. The coordination number and oxidation number X in the following compound $[X(SO_4)(NH_3)_5]Cl$ will be
(a) 10 and 3 (b) 02 and 6
(c) 06 and 3 (d) 06 and 4

30. In blast furnace, the highest temperature is in
(a) reduction zone (b) slag zone
(c) fusion zone (d) combustion zone

31. Atomic radii of which metals are same?
(a) Cr, Mn, Cu (b) Cr, Fe, Co
(c) Sc, Ti, Zn (d) V, Ni, Cr

32. The reagent which distinguishes formic acid and acetic acid is
(a) 2, 4-dinitrophenyl hydrazine
(b) $HgCl_2$
(c) C_2H_5ONa
(d) Hg_2Cl_2

33. If K_C is the equilibrium constant for the formation of NH_3, the dissociation constant of NH_3 under the same condition will be
(a) $\frac{1}{K_C}$ (b) K_C^2 (c) $\sqrt{K_C}$ (d) K_C

34. Neon atom requires energy to remove one electron from its outermost orbit, so also to add one electron into the outermost orbit.
Which of the following is correct about fluorine?
I. Fluorine releases energy when an electron is removed.
II. Fluorine requires energy to add one electron.
(a) (I) and (II) are correct
(b) (I) is correct, (II) is false
(c) (I) is false, (II) is correct
(d) (I) and (II) are false

35. In the metallurgy of iron, when limestone is added to the blast furnace, the calcium ions are removed as
(a) slag (b) gangue
(c) metallic Ca (d) $CaCO_3$

36. Liquid hydrocarbon is converted into mixture of gaseous hydrocarbons by
(a) cracking
(b) oxidation
(c) hydrolysis
(d) distillation under reduced pressure

37. Strongest bond among halogens is in
(a) F_2 (b) Cl_2 (c) Br_2 (d) I_2

38. Match List-I with List-II and select the correct answer from the given codes.

List - I (Reaction)	List - II (Reagent/Catalyst)
A. Cannizaro reaction	1. $SnCl_2$ / HCl
B. Stephen's reaction	2. NaOH
C. Clemmensen reduction	3. Zn / Hg—conc.HCl
D. Rosenmund's method	4. Pd / $BaSO_4$ / Boiling xylene

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	2	1	3	4
(c)	4	3	2	1
(d)	1	4	2	3

39. The vacant space in bcc unit cell is
(a) 10% (b) 23%
(c) 32% (d) 46%

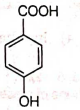
40. The pH of a soft drink is 3.92. The hydrogen ion concentration will be
(given antilog 0.08 = 1.2)
(a) 1.96×10^{-2} mol L^{-1} (b) 1.96×10^{-3} mol L^{-1}
(c) 1.2×10^{-4} mol L^{-1} (d) 1.2×10^{-3} mol L^{-1}

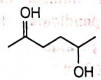
41. The correct order of ionic radius of nitrogen family is
(a) $N^{3-} < P^{3-} < As^{3-} < Sb^{3-} < Bi^{3-}$
(b) $N^{3-} < P^{3-} < Sb^{3-}$
(c) $P^{3-} = As^{3-} > Bi^{3-}$
(d) $N^{3-} > Bi^{3-} > Sb^{3-}$

42. On passing 0.5 Faraday of electricity through NaCl, the amount of Cl deposited on cathode is
(a) 35.5 g (b) 17.75 g
(c) 71 g (d) 142 g

43. Which of the following statement is false?
(a) Chlorophyll is responsible for the synthesis of carbohydrate in plants
(b) In presence of oxygen, haemoglobin forms oxyhaemoglobin
(c) Acetyl salicylic acid is known as aspirin
(d) Vitamin B_{12} contains Mg^{2+} ion

44. The correct order of increasing C—O bond length of CO, CO_2 and CO_3^{2-} is
(a) $CO_3^{2-} < CO_2 < CO$ (b) $CO_2 < CO_3^{2-} < CO$
(c) $CO < CO_3^{2-} < CO_2$ (d) $CO < CO_2 < CO_3^{2-}$

45. Among following, compound with the lowest pK_a value, is
(a) 
(b) 
(c) 
(d) 

46. Which one of the following will most readily be dehydrated in acidic solution?
(a) 
(b) 
(c) 
(d) 

47. Paracetamol is a following drug
(a) antipyretic
(b) antiseptic
(c) antibiotic
(d) anaesthetic

48. Which statement is not true for metals?
(a) They form bcc structure
(b) They form hcp structure
(c) They form ccp structure
(d) They have low enthalpy of sublimation

49. $K_2[HgI_4]$ detects the following ion
(a) Cl^- (b) NO_2^- (c) NO_3^- (d) NH_4^+

50. Which is the correct statement?
(a) Hydrogen peroxide can oxidise permanganate ion
(b) Permanganate ion can oxidise manganous ion
(c) Manganate ion is more stable than permanganate ion
(d) Permanganate ion cannot be reduced to manganate ion

Mathematics

1. The sum of the real solutions of equation $2|x|^2 + 51 = |1 + 20x|$ is
(a) 5 (b) 24
(c) 0 (d) None of these

2. The solution of the equation $\frac{dy}{dx} = \frac{1}{x+y+1}$ is
(a) $x + y = Ce^y - 2$ (b) $x + y = C\log y - 4$
(c) $\log(x+y+2) = Cy$ (d) $\log(x+y+2) = C + y$

3. The quadratic equation whose roots are $\frac{1}{3+\sqrt{2}}$ and $\frac{1}{3-\sqrt{2}}$, will be
(a) $7x^2 - 6x + 1 = 0$ (b) $6x^2 - 7x + 1 = 0$
(c) $x^2 - 6x + 7 = 0$ (d) $x^2 - 7x + 6 = 0$

4. Image point of (1, 3, 4) in the plane $2x - y + z + 3 = 0$ will be
(a) (3, 5, 2) (b) (3, 5, –2)
(c) (–3, 5, 2) (d) None of these

5. Who said, "Number of transistors per square inch on integrated circuits double every year, since their invention will continue to do so in the foreseable future"?
(a) Alan Turing (b) Jon Von Neumann
(c) Herbert Simon (d) Gorden Moore

6. The number of vectors of unit length perpendicular to vectors $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{b} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$, is
(a) infinite (b) one
(c) two (d) three

7. By trapezoidal rule, the approximate value of the integral $\int_0^6 \frac{dx}{1+x^2}$ is
(a) 1.3128 (b) 1.4108
(c) 1.4218 (d) None of these

8. The radius of a cylinder is increasing at the rate of 2 m/s and its height is decreasing at the rate of 3 m/s. When the radius is 3 m and height is 5 m, then the volume of the cylinder would change at the rate of
(a) 87π m³/s (b) 33π m³/s
(c) 27π m³/s (d) 15π m³/s

9. The point on the straight line $y = 2x + 11$ which is nearest to the circle $16(x^2 + y^2) + 32x - 8y - 50 = 0$, is
(a) $\left(\frac{9}{2}, 2\right)$ (b) $\left(\frac{9}{2}, -2\right)$
(c) $\left(-\frac{9}{2}, 2\right)$ (d) $\left(-\frac{9}{2}, -2\right)$

10. The locus of the extrimities of the latusrectum of the family of ellipses $b^2x^2 + y^2 = a^2b^2$ having a given major axis, is
(a) $x^2 \pm ay = a^2$
(b) $y^2 \pm bx = a^2$
(c) $x^2 \pm by = a^2$
(d) $y^2 \pm ax = b^2$

11. The solution of differential equation $(y\log x - 1)ydx = xdy$ is
(a) $y(\log e^x + Cx) = 1$ (b) $\left(\log\frac{x}{e} + Cx\right)x = y$
(c) $(\log Cx^2 + ex^2)y = x$ (d) None of these

12. If m things are distributed among a men and b women. Then, the chance that the number of things received by men is odd, is
(a) $\frac{(b-a)^m - (b+a)^m}{2(b+a)^m}$ (b) $\frac{(b+a)^m - (b-a)^m}{2(b+a)^m}$
(c) $\frac{(b+a)^m - (b-a)^m}{(b+a)^m}$ (d) None of these

13. The value of the integral $I = \int(\sqrt{\tan x} + \sqrt{\cot x})\,dx$, where $x \in \left(0, \frac{\pi}{2}\right)$, is
(a) $\sqrt{2}\sin^{-1}(\cos x - \sin x) + C$
(b) $\sqrt{2}\sin^{-1}(\sin x - \cos x) + C$
(c) $\sqrt{2}\sin^{-1}(\cos x + \sin x) + C$
(d) $-\sqrt{2}\sin^{-1}(\sin x + \cos x) + C$

14. The solution of the differential equation $\sqrt{a+x}\frac{dy}{dx} + xy = 0$ is
(a) $y = Ce^{\frac{2}{3}(2a-x)\sqrt{x+a}}$ (b) $y = Ce^{\frac{2}{3}(a-x)\sqrt{x+a}}$
(c) $y = Ce^{\frac{2}{3}(2a+x)\sqrt{x+a}}$ (d) $y = Ce^{-\frac{2}{3}(2a-x)\sqrt{x+a}}$

15. The values of a, if $f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$ increases x, are in
(a) $[0, \infty)$ (b) $(-\infty, 0]$
(c) $(-\infty, \infty)$ (d) $(1, \infty)$

16. By Newton-Raphson method, the positive root of the equation $x^4 - x - 10 = 0$ is
(a) 1.871 (b) 1.868
(c) 1.856 (d) None of these

17. If $\mathbf{a} = \frac{\hat{\mathbf{i}} - 2\hat{\mathbf{j}}}{\sqrt{5}}$ and $\mathbf{b} = \frac{2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{14}}$ are vectors in space, then the value of $(2\mathbf{a} + \mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} - 2\mathbf{b})]$ is
(a) 0 (b) 1
(c) 5 (d) 4

18. In ΔABC, if $3a = b + c$, then value of $\cot\frac{B}{2}\cot\frac{C}{2}$ will be
(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{2}$

19. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then value of x will be
(a) 0 (b) $\frac{1}{\sqrt{5}}$
(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

20. The value of $\int_0^{\sqrt{\ln(\pi/2)}} \cos(e^{x^2})2x\,e^{x^2}dx$ is
(a) 1 (b) 1 + sin 1
(c) 1 – sin 1 (d) (sin 1) –1

21. API stands for
(a) Access Programming Interface
(b) Android Programming Interface
(c) Application Programming Interface
(d) None of the above

22. Let A and B be two events of an experiment and $P(A) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$, then the value of $P\left(\frac{B}{A^C}\right)$ is
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{5}{6}$ (d) $\frac{1}{2}$

23. Distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = 0$ is
(a) 0 (b) 1
(c) 2 (d) 3

24. If $Z = i\log(2 - \sqrt{3})$, then the value of $\cos Z$ will be
(a) i (b) $2i$
(c) 1 (d) 2

25. If $2x - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$, then the value of x will be
(a) $\begin{bmatrix} 2 & 2 \\ 7 & 4 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 2 \\ 7/2 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} 2 & 2 \\ 7/2 & 1 \end{bmatrix}$
(d) None of the above

26. $\lim_{x \to 2} \frac{2 - \sqrt{2+x}}{2^{1/3} - (4-x)^{1/3}}$ is equal to
(a) $2 \cdot 3^{-1/2}$
(b) $3 \cdot 2^{-4/3}$
(c) $-3 \cdot 2^{-4/3}$
(d) None of the above

27. Two lines of regressions are given by $x + 2y - 5 = 0$ and $2x + ky - 8 = 0$. If $\sigma_x^2 = 12$ and $\sigma_y^2 = 4$, then the value of k is
(a) 1 (b) 2
(c) 3 (d) 4

28. The three lines of a triangle are given by $(x^2 - y^2)(2x + 3y - 6) = 0$. If the point $(-2, \lambda)$ lies inside and $(\mu, 1)$ lies outside the triangle, then

(a) $\lambda \in \left(1, \frac{10}{3}\right); \mu \in (-3, 5)$

(b) $\lambda \in \left(2, \frac{10}{3}\right); \mu \in (-1, 1)$

(c) $\lambda \in \left(-1, \frac{9}{2}\right); \mu \in \left(-2, \frac{10}{3}\right)$

(d) None of the above

29. System software : Utility software : :

(a) Operating system: Anti-virus

(b) Anti-virus : Operating system

(c) Anti-virus : MS office

(d) MS office : Anti-virus

30. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ is equal to

(a) 1

(b) 0

(c) −1

(d) Does not exist

31. The number of common tangents to two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 12 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

32. If $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g is the inverse of f. Then, the value of $g'(0)$ is

(a) 1

(b) 17

(c) $\sqrt{17}$

(d) None of the above

33. If $2a + 3b + 6c = 0$, then the equation $ax^2 + bx + c = 0$ has atleast one real root in

(a) $(0, 1)$

(b) $\left(0, \frac{1}{2}\right)$

(c) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(d) None of the above

34. If for all $x, y \in N$, there exists a function $f(x)$ satisfying $f(x + y) = f(x) \cdot f(y)$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, then value of n will be

(a) 4 (b) 5

(c) 6 (d) None of these

35. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

(a) 0 (b) 1

(c) 3 (d) −3

36. If $f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right), & \text{if } x < 1 \\ 3 - 2x, & \text{if } x \geq 1 \end{cases}$, then $f(x)$ has

(a) local minimum at $x = 1$

(b) local maximum at $x = 1$

(c) Both local maximum and local minimum at $x = 1$

(d) None of the above

37. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then

(a) $a^2 - b^2 + 2ac = 0$ (b) $(a - c)^2 = b^2 + c^2$

(c) $a^2 + b^2 - 2ac = 0$ (d) $a^2 + b^2 + 2ac = 0$

38. The general solution of the differential equation $\frac{dy}{dx} = y \tan x - y^2 \sec x$ is

(a) $\tan x = (C + \sec x)y$ (b) $\sec y = (C + \tan y)x$

(c) $\sec x = (C + \tan x)y$ (d) $\tan y = (C + \sec x)x$

39. $\int_0^{100} e^{x - [x]} dx$ is equal to

(a) $50(e - 1)$ (b) $75(e - 1)$

(c) $90(e - 1)$ (d) $100(e - 1)$

40. The values of λ such that $(x, y, z) \neq (0, 0, 0)$ and $(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(\hat{i}x + \hat{j}y + \hat{k}z)$ are

(a) 0, 1 (b) −1, 1

(c) −1, 0 (d) −2, 0

41. In ΔABC, if $\cot A$, $\cot B$ and $\cot C$ are in AP, then a^2, b^2 and c^2 are in

(a) HP

(b) AP

(c) GP

(d) None of the above

42. By Simpson's $\frac{1}{3}$rd rule, the approximate value of the integral $\int_1^2 e^{-x/2} dx$ using four intervals, is

(a) 0.377

(b) 0.487

(c) 0.477

(d) 0.387

43. A die is rolled three times. The probability of getting a larger number than the previous number is

(a) $\frac{5}{216}$ (b) $\frac{5}{54}$

(c) $\frac{1}{6}$ (d) $\frac{5}{36}$

44. A variable plane is at a constant distance p from the origin O and meets the axes at A, B and C. The locus of the centroid of the tetrahedron $OABC$ is

(a) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$

(b) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$

(c) $x^2 + y^2 + z^2 = 16p^2$

(d) $x^2 + y^2 + z^2 = p^2$

45. If $f(x) = \log_e(6 - |x^2 + x - 6|)$, then domain of $f(x)$ has how many integral values of x?

(a) 5 (b) 4

(c) Infinite (d) None of these

46. The graph of the equation $y^2 + z^2 = 0$ in three dimensional space is

(a) YZ-plane

(b) Z-axis

(c) Y-axis

(d) X-axis

47. If $Z_1 = 1 + i$, $Z_2 = -2 + 3i$ and $Z_3 = \frac{ai}{3}$ are collinear, where $i^2 = -1$, then the value of a will be

(a) −1 (b) 3

(c) 4 (d) 5

48. In how many ways, words can be made from the letters of the word BHARAT in which B and H are never together?

(a) 360 (b) 300

(c) 240 (d) 120

49. If a, b and c are in HP, then for any $n \in N$, which one of the following is true?

(a) $a^n + c^n < 2b^n$

(b) $a^n + c^n > 2b^n$

(c) $a^n + c^n = 2b^n$

(d) None of the above

50. The value of $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$ is

(a) $\frac{17}{7}$ (b) $\frac{-17}{7}$

(c) $\frac{7}{17}$ (d) $\frac{-7}{17}$

Answers

Physics

1. (a)	2. (c)	3. (a)	4. (a)	5. (a)	6. (a)	7. (d)	8. (c)	9. (a)	10. (c)
11. (c)	12. (a)	13. (b)	14. (a)	15. (d)	16. (b)	17. (b)	18. (b)	19. (*)	20. (b)
21. (b)	22. (d)	23. (b)	24. (c)	25. (b)	26. (c)	27. (d)	28. (a)	29. (b)	30. (d)
31. (b)	32. (b)	33. (c)	34. (b)	35. (*)	36. (d)	37. (d)	38. (a)	39. (d)	40. (a)
41. (c)	42. (a)	43. (b)	44. (a)	45. (d)	46. (d)	47. (a)	48. (a)	49. (d)	50. (a)

(*) None option is correct.

Chemistry

1. (b)	2. (a)	3. (a)	4. (b)	5. (b)	6. (c)	7. (d)	8. (a)	9. (c)	10. (a)
11. (d)	12. (c)	13. (d)	14. (b)	15. (c)	16. (c)	17. (b)	18. (a)	19. (b)	20. (a)
21. (d)	22. (d)	23. (a)	24. (c)	25. (a)	26. (b)	27. (d)	28. (b)	29. (c)	30. (d)
31. (a)	32. (b)	33. (a)	34. (d)	35. (a)	36. (b)	37. (b)	38. (b)	39. (c)	40. (c)
41. (a)	42. (b)	43. (d)	44. (d)	45. (a)	46. (a)	47. (a)	48. (d)	49. (d)	50. (b)

Mathematics

1. (d)	2. (d)	3. (a)	4. (c)	5. (d)	6. (c)	7. (b)	8. (b)	9. (c)	10. (a)
11. (d)	12. (b)	13. (b)	14. (a)	15. (a)	16. (a)	17. (c)	18. (b)	19. (b)	20. (c)
21. (c)	22. (b)	23. (b)	24. (d)	25. (c)	26. (c)	27. (c)	28. (d)	29. (a)	30. (d)
31. (c)	32. (c)	33. (a)	34. (a)	35. (c)	36. (b)	37. (a)	38. (c)	39. (d)	40. (c)
41. (b)	42. (c)	43. (b)	44. (b)	45. (d)	46. (a)	47. (d)	48. (b)	49. (b)	50. (d)

Solutions

Physics

1. As we know that,

$$166 \times 10^{-27}\ \text{kg} = 1\ \text{amu}$$

$$\therefore \quad 1\ \text{kg} = \frac{1}{166 \times 10^{-27}}\ \text{amu}$$

$$\therefore \quad 10 \times 10^{-3}\ \text{kg} = \frac{10 \times 10^{-3}}{166 \times 10^{-27}}\ \text{amu}$$

$$= \frac{1}{166} \times 10^{+27-2}\ \text{amu}$$

$$= \frac{10^{25}}{166}\ \text{amu}$$

Also we know that,

$$1\ \text{amu} = 931\ \text{MeV}$$

$$= 931 \times 16 \times 10^{-13}\ \text{J}$$

$$\therefore \quad \frac{10^{25}}{16}\ \text{amu} = \frac{931 \times 16 \times 10^{-13}}{16} \times 10^{25}\ \text{J}$$

$$= 8.97 \times 10^{14}\ \text{J} = 9 \times 10^{14}\ \text{J}$$

2. Warming of glass bulb due to filament is an example of heat transfer due to radiation.

3. The plate is symmetric about x and y-axes. Applying perpendicular axes theorem, we can write

$$I_0 = I_x + I_y = I + I \quad [\because I_x = I_y = I]$$

$$\Rightarrow \quad I_0 = 2I$$

4. Vertical component of earth's magnetic field is

$$B_V = B_H \tan\delta$$

where, B_H = horizontal component of earth's magnetic field

δ = angle of dip.

Total intensity of earth's magnetic field

$$B = \sqrt{B_H^2 + B_V^2}$$

$$= \sqrt{B_H^2 + B_H^2} \quad [\because \tan\delta = \tan 45° = 1]$$

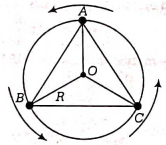
$$= \sqrt{2}\, B_H$$

$$= 1.44 \times 0.4 \times 10^{-4}\ \text{T}$$

$$= 5.656 \times 10^{-5}\ \text{T}$$

$$= 0.5 \times 10^{-4}\ \text{T}$$

5. Consider the movement of three bodies along a circle centered at O as shown in the diagram alongside.



Gravitational force between A and B

$$F_{BA} = \frac{Gm^2}{(AB)^2}$$

Gravitational force between B and C

$$F_{BC} = \frac{Gm^2}{(BC)^2}$$

From geometry of the figure, we can write

$BO \cos 30° + CO \cos 30° = BC$

$\Rightarrow \quad AB = BC = CA = 2R \cos 30° = \sqrt{3}R$

Net force on B,

$$F_B = F_{BA} \cos 30° + F_{BC} \cos 30°$$

$$= \frac{2Gm^2}{(AB)^2} \times \frac{\sqrt{3}}{2} = \frac{2Gm^2}{(\sqrt{3}R)^2} \times \frac{\sqrt{3}}{2} = \frac{2Gm^2}{3R^2} \times \frac{\sqrt{3}}{2}$$

This force will provide centripetal force for circular motion of B

$$F_B = \frac{2Gm^2}{3R^2} \times \frac{\sqrt{3}}{2} = \frac{mv_B^2}{R} = \frac{G}{\sqrt{3}R} = v_B^2$$

$$\Rightarrow \quad v_B = \sqrt{\frac{G}{\sqrt{3}R}}$$

6. Generalised formula for apparent frequency is

$$f_a = \left(\frac{v \pm v_o}{v \mp v_s}\right) f_0$$

According to the question source is stationary,

$$v_s = 0$$

$$f_a = f' = \left(\frac{v \pm v_o}{v}\right) f_0$$

Observer is moving towards the source

$$\Rightarrow \quad f_a = f' \left(\frac{v + v_o}{v}\right) f$$

where, v = speed of the wave

v_o = speed of the observer

f = original frequency of the source

7. Magnetic flux linked with the secondary due to current in the primary.

$$\phi_2 = MI_1$$

$$\Rightarrow \quad \frac{d\phi_2}{dt} = M\frac{dI_1}{dt}$$

$\Rightarrow$ Induced emf in the secondary

$$\varepsilon_2 = \frac{d\phi_2}{dt} = M\frac{dI_1}{dt}$$

where, M = mutual inductance = 15 H

$$\frac{dI_1}{dt} = \frac{5}{1\times10^{-3}}$$

$$= 5000\ \text{A/s}$$

$$\therefore \quad \varepsilon_2 = -15\times5000\ \text{V}$$

$$= 75\times10^3 V$$

$$= 7500\ \text{V}$$

8. Height or depth of liquid column in the capillary tube

$$y = \frac{2S\cos\theta}{r\delta g}$$

where, S = surface tension of the liquid

θ = angle of contact

r = radius of the tube

ρ = density of the liquid

For water,

$$h = \frac{2S_w\cos\theta}{r\rho_w g} \quad \text{... (i)}$$

For mercury,

$$d = \frac{2S_m\cos\theta}{r\rho_m g} \quad \text{... (ii)}$$

From Eqs. (i) and (ii),

$$\frac{h}{d} = \frac{S_w}{S_m}\times\frac{\rho_m}{\rho_w}\times\frac{\cos 0°}{\cos 135°}$$

$$\Rightarrow \quad \frac{10}{-3.42} = \frac{S_w}{S_m}\times\frac{13.6}{1}\times\frac{1}{\left(\frac{-1}{\sqrt{2}}\right)}$$

$$\Rightarrow, \quad \frac{10}{3.42} = \frac{S_w}{S_m}\times13.6\times\sqrt{2}$$

$$\text{or,} \quad \frac{S_w}{S_m} = \frac{10}{3.42}\times\frac{1}{13.6}\times\frac{1}{\sqrt{2}}$$

$$= 1:6.57$$

9. Given, $d = 128$ Å

Potential on the axis of a dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\times\frac{p}{r^2}$$

where, p = dipole moment

$$= qd = (1.6\times10^{-19})(12.8\ \text{Å})$$

$$r = 12\ \text{Å}$$

$$V = \frac{1}{4\pi\varepsilon_0}\times\frac{1.6\times10^{-19}\times(128\text{Å})}{(12\text{Å})^2}$$

$$= \frac{9\times10^9\times1.6\times10^{-19}\times128}{144\text{Å}}$$

$$= \frac{9\times10^{19}\times1.6\times10^{-19}\times128}{144}$$

$$= 10^{-1}\times128 = 0.128 \approx 0.13\ \text{V}$$

10. Magnetic field at the centre of a toroid is given by

$$B = \frac{\mu_0 Ni}{2\pi r}$$

where, $n = \frac{N}{2\pi r}$ = number of turns per unit length of the toroid.

11. Change of resistance with change of temperature is given by

$$R = R_0 \propto \Delta T$$

where, R_0 = resistance at certain temperature T_0.

$$R(T) - R(T_0) = R(T_0)\,\alpha\,[T - T_0]$$

$$\Rightarrow \quad R(T) = R(T_0)\,[1 + \alpha\,(T - T_0)]$$

12. Magnification produced by the lens

$$m = \frac{v}{u} = -3$$

$$\Rightarrow \quad v = -3u$$

Applying lens of formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-3u} - \frac{1}{u} = \frac{1}{0.12}$$

$$\text{or} \quad \frac{-1-3}{3u} = \frac{100}{12} = \frac{25}{3}$$

$$\text{or} \quad -\frac{4}{3u} = \frac{25}{3}$$

$$\text{or} \quad u = -\frac{4}{25}\text{m}$$

$$\Rightarrow \quad = 0.16\ \text{m}$$

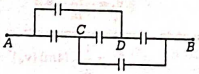
13. Rate of change of momentum of the object,

$$\frac{\Delta p}{\Delta t} = F_{ext} = \frac{mv^2}{r}$$

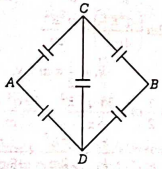
$$\frac{\Delta p}{\Delta t} \propto v^2 \quad \text{where } m \text{ and } r \text{ are constant}$$

14. Free electron density depends on the material of the wire and temperature. It is independent of area of cross-section of the current carrying wire.

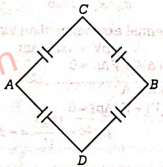
15. Consider the given circuit,



The circuit can be redrawn as shown below.



The circuit is a balanced bridge. We can remove the capacitor between C and D.



Thus, equivalent capacitance between A and B is

$$C_{AB} = \frac{C}{2} + \frac{C}{2} = C$$

16. Energy of an electron in nth state is

$$E_n = -13.6\times\frac{Z^2}{n^2}$$

$$= \frac{-13.6}{n^2}\ \text{eV}$$

[$\therefore Z = 1$, for hydrogen atom]

As given, $E_n = -3.4$ eV

$$\Rightarrow \quad -3.4\ \text{eV} = \frac{-13.6}{n^2}\ \text{eV}$$

$$\Rightarrow \quad n^2 = 4$$

$$\Rightarrow \quad n = 2$$

Angular momentum,

$$q = \frac{nh}{2\pi} = \frac{2\times h}{2\pi}$$

$$= \frac{h}{\pi}$$

$$= \frac{6.626\times10^{-34}}{3.14}\ \text{Js}$$

$$= 2.1\times10^{-34}\ \text{Js}$$

17. Electric field due to infinite plane sheets

$$|E_+| = |E_-| = \frac{\sigma}{2\varepsilon_0}$$

where, E_+ = electric field due to positively charged plane sheet

E_- = electric field due to negatively charged plane sheet.

Electric field between the plate

$$E = |E_+| + |E_-|$$

$$= 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

18. Time period of a suspended magnet is given by

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where, I = moment of inertia of the magnet about the point of suspension.

M = magnetic moment

B = magnetic field

When the magnet is broken in length

$$T^{2'} = 2\pi\sqrt{\frac{I'}{M'B}}$$

$$\Rightarrow \quad \frac{T}{T^{2'}} = \sqrt{\frac{I}{I'}}\times\sqrt{\frac{M'}{M}}$$

$$= \sqrt{\frac{mL^2}{m'L'^2}}\times\sqrt{\frac{L'}{L}}$$

Here, m and m' are the masses of the magnet and L and L's are their lengths.

$$= \sqrt{\frac{m}{m'}}\times\sqrt{\frac{L}{L'}}$$

$$= \sqrt{\frac{m}{m/2}}\times\sqrt{\frac{L}{L/2}}$$

$$= \sqrt{2}\times\sqrt{2} = 2$$

$$\Rightarrow \quad T' = \frac{T}{2} = \frac{4}{2} = 2\ \text{s}$$

19. $v_1 = 1\ L = 10^3$ cc

$$p_1 = 72\ \text{cm of Hg}$$

$$= 0.72\ \text{m of Hg}$$

$$= 0.72\times13.6\times10^3\times10$$

$$= 9.7\times10^4\ \text{N/m}^2$$

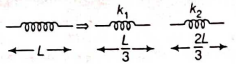
We have $p_1V_1 = p_2V_2$

$$p_2 = \frac{p_1V_1}{V_2}$$

$$= \frac{9.7\times10^4\times10^3}{900}$$

$$= 10.8\times10^4\text{N/m}^2$$

20. Let length of the original spring is L.



As we know that,

Spring constant, $k \propto \frac{1}{\text{length}}$

$$\Rightarrow \quad k \propto \frac{1}{L}$$

$$k_1 \propto \frac{3}{L}$$

$$k_2 \propto \frac{3}{2L}$$

$$\Rightarrow \quad \frac{k_2}{k} = \frac{3/2L}{1/L} = \frac{3}{2}$$

$$\Rightarrow \quad k_2 = \frac{3}{2}k$$

21. Average value of output voltage in a half wave rectifier is

$$V_{av} = \frac{V_0}{\pi}$$

Average value of current

$$I_{av} = \frac{V_{av}}{R} = \frac{V_0}{R\pi}$$

$$= \frac{I_0}{\pi}$$

where, R is resistance.

22. Number of half-lives in 40 min

$$n = \frac{40}{T_{1/2}} = \frac{40}{10} = 4$$

Amount of the substance remaining after n half- lives

$$N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

where, N_0 is original amount of the substance.

Amount of the substance decayed in 40 min.

$$N' = N_0 - N = N_0 - \frac{N_0}{16} = \frac{15}{16}N_0$$

$$= \left(\frac{15}{16} \times 100\%\right) \text{ of } N_0$$

$$= 15 \times 6.25\ \% \text{ of } N_0$$

$$= 93.75\ \% \text{ of } N_0$$

23. Velocity acquired in 2 s. by the charged particle of mass 4 m,

$$v_4 = at = \frac{q(E)}{4m} \times 2$$

where, E is magnitude of electric field

Similarly,

$$v_1 = \frac{3q(E)}{m} \times 2$$

Corresponding kinetic energies are

$$k_4 = \frac{1}{2} \times (4m)(v_4)^4$$

$$= \frac{1}{2} \times (4m)\left(\frac{qE}{2m}\right)^2$$

$$2m\left(\frac{q^2E^2}{4m^2}\right) = \frac{q^2E^2}{2m}$$

$$k_1 = \frac{1}{2} \times m \times v_1$$

$$= \frac{1}{2}m \times \frac{36\,q^2\,E^2}{m^2} = \frac{18\,q^2\,E^2}{m}$$

$$\Rightarrow \quad \frac{k_4}{k_1} = \frac{q^2E^2}{2m} \times \frac{m}{18\,q^2E^2}$$

$$= \frac{1}{36}$$

or, $\quad k_4 : k_1 = 1:36$

24. For isothermal expansion of an ideal gas

$$pV = \text{constant}$$

$$\Rightarrow \quad p\,\Delta V + V\Delta p = 0$$

For container A,

$$p_{iA}\,(V) + V\,(\Delta p) = 0$$

$$\Rightarrow \quad p_{iA} = -\frac{V\Delta P}{V} = -\Delta p \qquad \text{...(i)}$$

where, p_i is initial pressure.

For container B,

$$p_{iB}\,(V) + V\,(15\,\Delta p) = V$$

$$\Rightarrow \quad p_{iB} = -15\,\Delta p \qquad \text{.....(ii)}$$

From Eqs. (i) and (ii), we have

$$p_{iA} = \frac{p_{iB}}{15} = \frac{2}{3}p_{iB}$$

$$\Rightarrow \quad m_A = \frac{2}{3}mB$$

$$\Rightarrow \quad 3m_A = 2mB \qquad [\because p \propto m]$$

25. For monoatomic gas,

$$\gamma_1 = \frac{C_{P_1}}{C_{V_1}} = \frac{5}{3}$$

and $\quad C_{P_1} - C_{V_1} = R$

this gives $C_{P_1} = \frac{5}{2}R$ and $C_{V_1} = \frac{3}{2}R$

For diatomic gas,

$$\gamma_2 = \frac{C_{P_2}}{C_{V_2}} = 7/5$$

and $\quad C_{P_2} - C_{V_2} = R$

$$\Rightarrow \quad C_{P_2} = \frac{7}{2}R$$

and $\quad C_{V_2} = \frac{5}{2}R$

Now, we can write,

$$\gamma_{mix} = \frac{n_1\,C_{P_1} + n_2\,C_{P_2}}{n_1\,C_{V_1} + n_2\,C_{V_2}}$$

$$= \frac{C_{P_1} + C_{P_2}}{C_{V_1} + C_{V_2}} = \frac{\frac{5}{2}R + \frac{7}{2}R}{\frac{3}{2}R + \frac{5}{2}R}$$

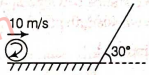
$$= \frac{12}{8}$$

$$= \frac{3}{2} = 15$$

26. The correct relation between specific heat capacities of gases C_p and C_V is

$$C_p - C_V = R$$

27. Applying conservation of mechanical energy, we can write



$$\frac{1}{2}I\,\omega^2 + \frac{1}{2}mv^2 = mgh$$

where, h is maximum height reached by the sphere, v is linear velocity and ω is angular velocity of the sphere.

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh \quad [\because v = \omega r]$$

$$\Rightarrow \quad \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{v^2}{r^2} + \frac{1}{2}mv^2 = mgh$$

$$\frac{1}{5}mv^2 + \frac{1}{2}mv^2 = mgh$$

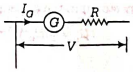
$$\frac{7}{10}mv^2 = mgh$$

$$h = \frac{7}{10} \times \frac{v^2}{g}$$

$$= \frac{7}{10} \times \frac{10^2}{98} = \frac{70}{98}$$

$$= \frac{100}{14} = \frac{50}{7} = 7.1 \text{ m.}$$

28. To convert the galvanometer to a voltmeter, the resistance must be connected in series. Suppose the resistance is R,



The potential difference is

$$V = I_G\,R_G + I_G R$$

$$\Rightarrow \quad 50 = 5 \times 10^{-3} \times 1000 + 5 \times 10^{-3} \times R$$

$$\Rightarrow \quad 50 = 5 + 5 \times 10^{-3}\,R$$

$$\Rightarrow \quad 45 = 5 \times 10^{-3}\,R$$

$$\Rightarrow \quad R = 9000\ \Omega$$

29. Suppose depth inside the earth is h.

As we know that, inside the earth

$$g_i \propto \left(\frac{R-h}{R^3}\right) \qquad \text{.....(i)}$$

On the surface of the earth

$$g_s \propto \frac{1}{R^2} \qquad \text{.....(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{g_i}{g_s} = \frac{(R-h)\,R^2}{R^3} = \frac{R-h}{R}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{R-h}{R}$$

$$\Rightarrow \quad R = 4R - 4h$$

$$\Rightarrow \quad 3R = 4h$$

$$\Rightarrow \quad h = \frac{3}{4}R.$$

30. Magnitude of the magnetic moment of the coil is

$$M = nIA$$

where, n = number of turns

I = current in the coil

A = area of circular coil.

$$\therefore \quad M = 300 \times 15 \times \pi\,(7 \times 10^{-2})^2$$

$$= 300 \times 15 \times \frac{22}{7} \times 49 \times 10^{-4}$$

$$= 3 \times 15 \times 22 \times 7 \times 10^{-2}$$

$$= 6930 \times 10^{-2} = 69.3 \approx 70 \text{ J/T}.$$

31. According to the question

Work function $\phi = 3.3$ eV

$$\Rightarrow \quad h\,\nu_0 = 3.3 \text{ eV}$$

where, ν_0 = threshold frequency

h = Planck's constant

$$\therefore \quad 6.626 \times 10^{-34} \times \nu_0 = 3.3 \times 16 \times 10^{-19}$$

$$\text{or,} \quad \nu_0 = \frac{3.3 \times 16 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$= \frac{5.28}{6.626} \times 10^{15}$$

$$= \frac{5.28}{6.626} \times 10^{14} = 8 \times 10^{14} \text{ Hz}$$

32. In the given equation, S_n is the displacement in nth beond.

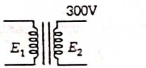
Thus, dimension of S_n is same as length

i.e. $[S_n] = [L] = [M^0 L^1 T^0]$

33. Range of a projectile is maximum for angle of projection of 45°.

The nearest value is 43°. Thus, among given values of angles of projection, range will be maximum for 43°.

34. For the transformer shown in the figure, we can write



$N_1 = 200$ $150 = N_2$

Primary Secondary

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\Rightarrow \quad \frac{E_1}{300} = \frac{200}{150} \Rightarrow E_1 = 400 \text{ V}.$$

35. Resistance of a wire is given by

$$R = \frac{\rho L}{A}$$

where, ρ = resistivity

L = length of the wire

A = area of cross- section of the wire

Equating initial and finial volumes of the wire, we can write

$$AL = A'L' = A' \times (2L)$$

$$\Rightarrow \quad A' = \frac{A}{2}$$

$$\therefore \quad \frac{R'}{R} = \frac{\rho L'/A'}{\rho L/A} = \left(\frac{L'}{L}\right)\left(\frac{A}{A'}\right)$$

$$= \left(\frac{2L}{L}\right)\left(\frac{A}{A/2}\right) = 4$$

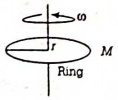
$$\Rightarrow \quad R' = 4R = 4(5) = 20\,\Omega$$

36. Conserving angular momentum, we can write

$$L_i = L_f$$

where, L_i = initial angular momentum of the ring

L_f = angular momentum of the (ring + mass) system.



$$\therefore \quad I_i\,\omega_i = I_f\,\omega_f$$

or $$Mr^2\,\omega = [2mr^2 + Mr^2]\,\omega_f$$

or $$\omega_f = \frac{\omega M}{M + 2m}$$

37. Magnifying power of a telescope is given by $m = \frac{f_o}{f_e}$

If the focal length of the eye piece is doubled, the new magnifying power is given by

$$m' = \frac{f_o}{f_e'} = \frac{f_o}{2f_e}$$

$$\therefore \quad m' = \frac{m}{2}$$

38. Fringe width in Young's double slit experiment is given by

$$\beta = \frac{D}{d}\lambda \quad \text{.....(i)}$$

where, D = distance of the screen from the slits.

d = distance between S_1 and S_2.

λ = wavelength of light.

When d is reduced

$$\beta' = \frac{D}{d'}\lambda \quad \text{.....(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\beta}{\beta'} = \frac{d'}{d} < 1 \quad [\because d' < d]$$

$$\Rightarrow \quad \beta' > \beta$$

Thus, we can say that fringe width increases.

39. Given $V = 250$ V and $P = 500$ W

As we know that,

$$P = \frac{V^2}{R}$$

$$\Rightarrow \quad R = V^2/P = \frac{250 \times 250}{500}$$

$$= \frac{625}{5} = 125\,\Omega$$

40. As the battery remains connected. potential difference (V) of the capacitor remains constant. Suppose dielectric of dielectric constant K is introduced between the plates. Now, capcitance of the capacitor is

$$C' = KC_0$$

where, C_0 is original capacitance.

Initial energy stored in the capacitor

$$U_i = \frac{1}{2}C_0 V^2$$

Final energy stored in the capacitor

$$U_f = \frac{1}{2}C' V^2 = \frac{1}{2}(K C_0) V^2$$

Clearly, $$\frac{U_f}{U_i} = K > 1$$

$$\Rightarrow \quad U_f > U_i$$

As, $Q = CV$

$\therefore$ V remains constant and C increases.

$\therefore$ Q increases .

41. Work done in stretching a wire by Δx

$\Delta W = \Delta V$ = potential energy stored in the wire

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times Y \times \text{strain} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times Y \times (\text{strain})^2 \times (AL)$$

$$= \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 \times (AL)$$

where, symbols have their usual meanings.

For first wire,

$$\Delta W_1 = \frac{1}{2} \times Y \left(\frac{\Delta L_1}{L_1}\right)^2 \times (A_1 L_1)$$

For second wire,

$$\Delta W_2 = \frac{1}{2} \times Y \left(\frac{\Delta L_2}{L_2}\right)^2 \times (A_2 L_2)$$

Thus we can write,

$$\frac{\Delta W_2}{\Delta W_1} = \left(\frac{\Delta L_2}{\Delta L_1}\right)^2 \times \left(\frac{L_1}{L_2}\right)^2 \times \left(\frac{A_2 L_2}{A_1 L_1}\right)$$

$$= \left(\frac{1\text{mm}}{1\text{mm}}\right)^2 \times \left(\frac{L}{L/2}\right)^2 \times \left(\frac{L/2}{2}\right) \times \left(\frac{2r}{r}\right)^2$$

$$= 1 \times 4 \times \frac{1}{2} \times 4 = 8$$

$$\Rightarrow \quad \Delta W_2 = 8(\Delta W_1) = 8(2\text{J}) = 16 \text{ J}$$

42. Power of the combination

$$P = P_1 + P_2$$

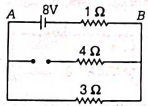
$$= P_{\text{concave}} + P_{\text{convex}}$$

$$\Rightarrow \quad \frac{100}{50} = P_{\text{concave}} + \frac{100}{20}$$

or $$2 = P_{\text{concave}} + 5$$

or $$P_{\text{concave}} = 2 - 5 = -3.0 \text{ D}$$

43. In steady state, the capacitor will act as open circuit.



Current through the circuit

$$I = \frac{8}{3+1} = 2\text{A}.$$

Voltage across AB,

$$V_{AB} = I \times 3 = 2 \times 3 = 6\text{V}$$

Voltage across the capacitor $V_c = V_{AB} = 6\text{V}$

Charge on the capacitor

$$Q = CV_c = 2 \times 10^{-6} \times 6 = 12\,\mu\text{C}$$

44. Critical angle (θ_c) for glass water interface can be written as

$$\sin\theta_c = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

$$\Rightarrow \quad \theta_c = \sin^{-1}(8/9)$$

45. Path difference between the waves reaching third bright fringe and central fringe is

$$\Delta x = 3\lambda$$

As we know that, path difference of λ corresponds to phase difference of 2π.

$\therefore$ Δx is equivalent to phase difference of $\Delta\phi$

$$= 3(2\pi) = 6\pi$$

46. Power radiated by a black body is given by

$$P = \sigma A T^4$$

where, σ = Stefan's constant

T = absolute temparature

A = surface area

$$P_1 = \sigma A_1 T_1^4 \quad \text{...(i)}$$

$$P_2 = \sigma A_2 T_2^4 \quad \text{...(ii)}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{A_1}{A_2}\right)\left(\frac{T_1}{T_2}\right)^4$$

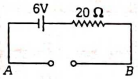
or, $$\frac{450}{P_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{500}{2 \times 500}\right)^4$$

$$= \left(\frac{r}{r/2}\right)^2 \left(\frac{1}{2}\right)^4$$

$$= 4 \times \frac{1}{16} = \frac{1}{4}$$

or, $$P_2 = 4 \times 450 = 1800 \text{ W}$$

47. In the given circuit, the diode is reverse biased. Only drift current flows through the diode.



Consider the circuit in which by applying KVL, we can write

$$V_A - 6 + I \times 20 = V_B$$

$$\Rightarrow V_A - V_B = 6 - I \times 20 = 6 - 30 \times 10^{-6} \times 20$$

$$= 6 - 0.006$$

$$= -5.9994\text{V}$$

48. According to the question, given that

$$U(x) = k|x|^3$$

Force on the particle,

$$F = \frac{-dU}{dx} = \frac{-d}{dx}[kx^3]$$

$$= -3kx^2$$

At x = amplitude (a)

$$F = |-3ka^2| = m\omega^2 a \qquad [\because k' = m\omega^2]$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3ka}{m}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3ka}}$$

$\Rightarrow$ Time period,

$$T \propto \frac{1}{\sqrt{a}}$$

49. Given, power $P = 3\text{D}$

$$f = \frac{1}{P} = \frac{1}{3}\text{m} = \frac{100}{3}\text{cm}.$$

$$u = -25\text{cm}.$$

Applying lens formula, we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{25} = \frac{3}{100}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{3}{100} - \frac{1}{25} = \frac{3-4}{100} = \frac{-1}{100}$$

$$v = -100\text{ cm}.$$

Now, again using lens formula we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

As, focal length of a far sighted person deceases

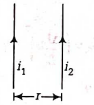
$$\Rightarrow \quad \frac{1}{-100} - \frac{1}{u} > \frac{1}{17} = \frac{10}{17} \qquad [\because f < 17\text{ cm}]$$

$$\Rightarrow \quad \frac{1}{u} < \frac{-1}{100} - \frac{10}{17}$$

$$= \frac{-17-1000}{1700} = \frac{-1017}{1700}$$

$$\Rightarrow \quad u > \frac{1700}{1000} \approx -1.7 = -1\text{m} \qquad \text{(nearest option)}$$

50. Force per unit length between the two parallel current carrying wires



$$\frac{F}{L} = \frac{\mu_0}{2\pi r} i_1 i_2$$

$$= \frac{\mu_0}{2\pi(2d)} i_1 i_2 = \frac{\mu_0 i_1 i_2}{4\pi d} \qquad [\because r = 2d]$$

Chemistry

1. (+)- Sucrose $\xrightarrow{\text{HOH / H}^+}$ D (+) - glucose + D (−) - fructose

($C_{12}H_{22}O_{11}$) ($C_6H_{12}O_6$) ($C_6H_{12}O_6$)

Maltose $\xrightarrow{\text{HOH / H}^+}$ Glucose + Glucose

($C_{12}H_{22}O_{11}$)

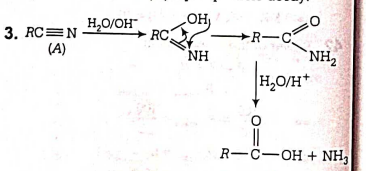
Lactose $\xrightarrow{\text{HOH / H}^+}$ D (+)-glucose + D (+)- galactose

($C_{12}H_{22}O_{11}$)

Hence, A = Sucrose, B = Maltose, C = Lactose

2. A decay process is possible only if accompanied by the release of energy. There are three different decay processes that increase the n/p ratio. These are : (i) electron capture, (ii) position emission (β^+- emission) and (iii) alpha particle decay.

3.



It is a very good synthetic method for the hydrolysis of cyanides with acid or alkali.

4. Kerosene is a thin, clear liquid formed from hydro carbons obtained from the fractional distillation of petroleum between 150-275°C resulting in a mixture with a density of 0.78-0.81 g/cm^3 composed of carbon chains that typically contain between 6 and 16 carbon atoms per molecule.

5. Gasoline mainly has compounds from C_5-C_{10} range. Gasoline is used in spark-ignition (SI) engines. Octane number is a measure of the anti-knocking capacity of the fuel. 2, 2, 4 - trimethylpentane (*Iso*-octane) has been designated an octane number of 100 and *n*-heptane has been assigned an octane number of zero.

This sample of gasoline contain 81% octane. Hence, it's octane number will be 81.

6. Poly tetrafluoro ethylene (PTFE) or teflon $-(CF_2-CF_2)_n-$ is an addition homopolymer. It is used as lubricant, insulator and in making cooking wares.

7. $\Delta G = \Delta H - T\Delta S$

At constant pressure

$$\Delta H = \Delta H_{vap}$$

$$\Delta s = \frac{\Delta q_{rev}}{\Delta T}$$

and $$\Delta S = \frac{1}{T}\int dq_{rev} = \frac{\Delta H_{vap}}{T}$$

and $$T\Delta S = \Delta H_{vap}$$

Hence, $$\Delta G = \Delta H_{vap} - H_{vap} = 0$$

The isothermal and isobaric reversible conversion of water into vapour involves no change in free energy.

8. According to Raoult's law, relative lowering in vapour pressure

$$\frac{p_A^\circ - p_S}{p_A^\circ} = \frac{n_B}{n_A + n_B}$$

$$n_A = \frac{180}{18} = 10$$

and $$n_B = \frac{3.42}{342} = 0.01$$

Here, atmospheric pressure p_A° = 760 mm of Hg

$$\therefore \quad \frac{760 - p_S}{760} \quad p_S = \frac{0.01}{10 + 0.01}$$

$$= \frac{0.01}{10.01}$$

$$= 0.00099$$

or, $$760 - p_S = 0.75924$$

or, $$p_S = 760 - 0.75924$$

$$= 759.24076\text{ mm}$$

9. Isoelectronic species have same number of electrons.

$$CO = 6 + 8 = 14$$

$$NO^+ = 7 + 8 - 1 = 14$$

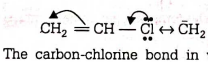
$$CN^- = 6 + 7 + 1 = 14$$

$$C_2^{2-} = 2 \times 6 + 2 = 14$$

10. The IUPAC name of the given compound is 3-methyl cyclohexene.

11. The correct representation of a complexionian is $[Co(NH_3)_4(H_2O)Cl]^{2+}$.

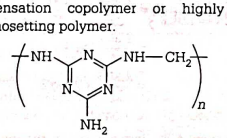
12. The non-reactivity of chlorine atom in vinyl chloride is due to resonance stabilisation. The lone pair on chlorine can participate in delocalisation (resonance) to give two cannonical structures.



The carbon-chlorine bond in vinyl chloride has some double bond character and is therefore stronger than a pure single bond.

13. Atoms and ions having inert gas configuration (such as helium), i.e. stable (half-filled) configuration have high ionisation potential, i.e. it is relatively easy to remove an electron from a partially filled valence shell, where Z_{eff} is lower but it is relatively difficult to remove an electron from an atom or ion that has a filled valence shell, where Z_{eff} is higher.

14. Melamine-formaldehyde resin or Melmac is a condensation copolymer or highly branced thermosetting polymer.



15. $\Delta H = \Delta E + \Delta n_g RT$

where Δn_g = number of moles of gaseous product – number of moles of gaseous reactant

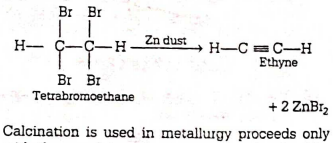
For $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

$$\Delta n_g = 2 - 2 = 0$$

Thus, $$\Delta H = \Delta E + 0 \times RT$$

or, $$\Delta H = \Delta E$$

16. Dehalogenation of tetrabromoethane with Zn dust gives ethyne.



17. Calcination is used in metallurgy proceeds only with the expulsion of some small molecules like water, CO_2, SO_2 etc. without any chemical changes. e.g.

$$2Al(OH)_3 \xrightarrow{1500°C} Al_2O_3 + 3H_2O\uparrow$$

$$FeCO_3 \xrightarrow{\Delta} FeO + CO_2\uparrow$$

18. Empirical formula mass of $CH_2 = 12 + 2 = 14$

Molecular mass = 42

$\because$ $n = \dfrac{\text{Molecular mass}}{\text{Empirical formula mass}}$

$\therefore$ $n = \dfrac{42}{14} = 3$

Then, molecular formula = (empirical formula)

$= (CH_2)_3$

$= C_3H_6$

19. $[Ni(CN)_4]^x \equiv [Ni(CN)_4]^{2-}$

In this complex, the oxidation state of nickel is + 2 and that of (CN) is − 1.

$\therefore$ $x = +2 + y(-1) = -2$

20. For first order reaction,

$$k = \frac{2.303}{t}\log_{10}\frac{a}{(a-x)}$$

or, $$k = \frac{2.303}{32}\log_{10}\frac{100}{(100-75)}$$

or, $$k = \frac{2.303}{32}\log\frac{100}{25}$$

$$= \frac{2.303}{32}\log 4$$

$$= \frac{2.303}{32} \times 2\log 2$$

$$= 0.0433$$

Reaction 50% completed, it means that time required for 50% completion equal to $t_{1/2}$.

$\therefore$ $k = \dfrac{0.693}{t_{1/2}}$

$\therefore$ $t_{1/2} = \dfrac{0.693}{k}$

$= \dfrac{0.693}{0.0433}$

$= 16.004$ min

21. From Arrhenius equation,

$$\log k = \log A - \frac{E_a}{2.303\,RT}$$

As the activation energy of a reaction is zero, the rate constant of this reaction becomes zero. It shows that the reaction is independent of temperature.

22. Mercury (II) chloride or mercuric chloride (archaically, corrosive sublimate) is the highly corrosive in nature, chemical compound of mercury with chlorine with the formula $HgCl_2$.

23. Given, concentration of $SrCl_2 = 10^{-3}$ mol% concentration is in percentage so that take total 100 mL of solution.

Number of moles of NaCl $= 100^{-3}$ moles of $SrCl_2$

Moles of $SrCl_2$ is very negligible as compare to total moles so percentage always taken as 100 so that

1 mole of NaCl is dipped with $= \dfrac{10^{-3}}{100}$ mole of $SrCl_2$

$= 10^{-5}$ mole of $SrCl_2$

So, cation vacancies per mole of NaCl $= 10^{-6}$ mol

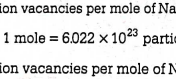
Since, 1 mole $= 6.022 \times 10^{23}$ particles

So, cation vacancies per mole of NaCl

$= 10^{-5} \times 6.022 \times 10^{23}$

$= 6.022 \times 10^{18}$

24. I. $H_2S_3O_6$– trithionic acid is an example of polythionic acid.



III. H_2S_4 or HS—S—S—SH is an hydrosulfane.

The compounds (II) and (IV) do not exist.

25. Given, KE $= 4.55 \times 10^{-25}$ J

$\because$ $KE = \dfrac{1}{2}mv^2$

$\therefore$ $4.55 \times 10^{-25} = \dfrac{1}{2} \times 9.1 \times 10^{-31} \times v^2$

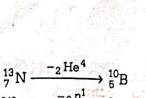
(mass of electron $= 9.1 \times 10^{-31}$ kg)

$$\frac{4.55 \times 10^{-25} \times 2}{9.1 \times 10^{-31}} = v^2$$

or, $v^2 = 10^6$

or, $v = \sqrt{10^6} = 10^3$ m s^{-1}

26.



27. Catalyst increases (or decreases) rate of both forward and backward reactions equally, therefore equilibrium will be attained in less (or more) time. So, only the time required to reach the equilibrium state is changed by the presence of catalyst. Thus, catalyst does not affect equilibrium constant.

28. When cold H_2O_2 is added in cold mixture of $K_2Cr_2O_7$ and conc. H_2SO_4 (i.e. $K_2Cr_2O_7$ in acidic medium), a deep blue solution of blue perchromate is obtained as,

$$K_2Cr_2O_7 + H_2SO_4 \longrightarrow K_2SO_4 + H_2Cr_2O_7$$

$$H_2Cr_2O_7 + 4H_2O_2 \longrightarrow \underset{\text{Blue perchromate}}{2CrO_5} + 5H_2O$$

29. $[x(SO_4)(NH_3)_5]Cl = [Co\underbrace{(SO_4)(NH_3)_5}_{\text{Ligands}}]Cl$

Let the oxidation state of Co is x.

$x + (-2) + 5 \times 0 + 1 \times (-1) = 0$

or $x - 2 + 0 - 1 = 0$

or $x = +3$

Hence, coordination number = 6

Oxidation number = 3

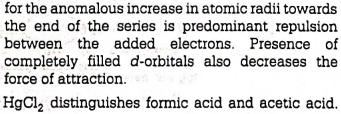
30. In blast furnace the highest temperature is in combustion zone. In the first stage, coke burns to produce CO_2 which rises temperature.

$$C + O_2 \longrightarrow CO_2 + \text{Heat}$$

This reaction is exothermic and hence temperature is 2170 K. It is known as combustion zone.

31. The atomic radii of elements from Cr to Cu (i.e. Cr, Mn, Cu) are very close to one another. This is due to effective screening of outer *ns*-electrons by the electrons of $(n-1)$ *d* subshell. The reason for the anomalous increase in atomic radii towards the end of the series is predominant repulsion between the added electrons. Presence of completely filled *d*-orbitals also decreases the force of attraction.

32. $HgCl_2$ distinguishes formic acid and acetic acid. Formic acid produces a white precipitate with $HgCl_2$ solution. Acetic acid does not give this test.



33. $N_2 + 3H_2 \underset{K_C'}{\overset{K_C}{\rightleftharpoons}} 2NH_3$

If the equation representing equilibrium constant is reversed, the new equilibrium constant is inverse of the first value, i.e. $K_C' = \dfrac{1}{K_C}$

34. (i) Fluorine required energy when an electron is removed, i.e. ionisation energy.

(ii) Fluorine releases energy to add one electron, i.e. electron affinity (− 328 kJ mol^{-1})

35. In the metallurgy of iron, when limestone is added to the blast furnace, the calcium ions are removed as slag.

$$CaCO_3 \xrightarrow{\Delta} CaO + CO_2$$

$$CaO + SiO_2 \longrightarrow \underset{\text{Calcium silicate (slag)}}{CaSiO_3}$$

36. Liquid hydrocarbon is converted into mixture of gaseous hydrocarbons by oxidation oxidation can be complete or partial. Complete oxidation is called combustion which is shown by the generalised reaction as

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \xrightarrow{\text{Flame}} nCO_2 + (n+1)H_2O + \text{Heat}$$

37. Strongest bond among halogens is in Cl_2. As the bond is strong, their bond dissociation energy is high. With the increase of size, the bond length increases from fluorine to iodine. Since, the bond length of fluorine is minimum, its bond dissociation energy should be highest. However, the bond dissociation energy of fluonne is less than Cl—Cl and Br—Br.

Bond energy	F	Cl	Br	I
Value (in kJ mol^{-1})	158.8	242.6	192.8	151.1

38. A → 2 B → 1 C → 3 D → 4

Cannizaro reaction – NaOH

Stephan's reaction – $SnCl_2$/HCl

Clemmensen reduction – Zn/Hg –Conc. HCl

Rosenmund's method – $\dfrac{\text{Pd-BaSO}_4}{\text{Boiling xylene}}$

39. In body centred cubic structure,

$$\text{Packing fraction} = \frac{\frac{8}{3}\pi r^3}{\frac{64\,r^3}{3\sqrt{3}}} \times 100 = 68\%$$

$\therefore$ Fraction of empty space = 100 − 68 = 32%

40. pH = 3.92

$\because$ $pH = -\log[H^+]$

$\therefore$ $3.92 = -\log[H^+]$ or $-3.92 = \log[H^+]$

or $[H^+] = \text{antilog } \bar{4}.08$ or, $[H^+] = 1.2 \times 10^{-4}$

41. Atomic raddii as well as ionic radii increases down the group. All the element of the nitrogen family contain same charge, i.e. – 3.

Thus, the correct order of ionic radius of nitrogen family is $N^{3-} < P^{3-} < As^{3-} < Sb^{3-} < Bi^{3-}$

42. $NaCl \rightleftharpoons Na^+ + \underset{(35.5\,g)}{Cl^-}$

$\because$ 1 Faraday of electricity deposited = 35.5 gm Cl on catholde

$\therefore$ 0.5 Faraday of electricity will deposited

$= 0.5 \times 35.5 = 17.75$ g Cl

43. Chlorophyll is responsible for the synthesis of glucose (carbohydrate) in plants.

In presence of oxygen, haemoglobin forms oxyhemoglobin.

Acetyl salicylic acid is known as aspirin

Vitamin B_{12} (Cyanocobalamin, $C_{63}H_{88}O_4N_{14}PCo$) contains Co^{2+} ion.

44. Bond length $\propto \dfrac{1}{\text{Bond order}}$

Bond order in CO = 3 (By molecular orbital theory)

Bond order in CO_2

$= \dfrac{\text{Number of bonds in all possible sides}}{\text{Number of resonating structures}}$

$= \dfrac{4}{2} = 2$

Bond order in $CO_3^{2-} = \dfrac{4}{3} = 1.33$

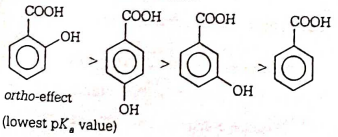
$\therefore$ Order of bond order is

$CO_3^{2-} < CO_2 < CO$

So, order of bond length of C—O will be

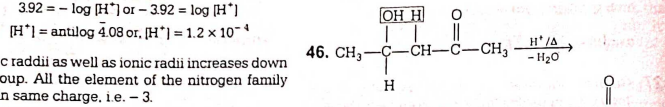
$CO < CO_2 < CO_3^{2-}$

45. Strength of acid is indicated by pK_a value. Higher the value of K_a or lower the value of pK_a, stronger is the acid. Among the given aromatic acids, the strength decreases as follows



***Ortho*-effect** The effect of a group is maximum at *ortho*-position due to nearness, is called *ortho* effect.

46.



Among the given compounds, the compound present in option (a) is readily dehydrated in acidic medium due to presence of acidic hydrogen with —OH group, In option (d), the strength of acidic hydrogen is less.

47. The chemical substances which are used to bring down body temperature during high fever are called antipyretics, e.g. paracetamol, aspirin, phenacetin etc.

Paracetamol
(4-acetamidophenol)

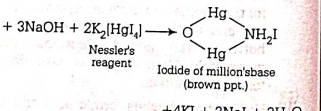
48. Enthalpy of sublimation is the enthalpy change when one mole of a solid sublimes (i.e. converts directly into gaseous state) at a temperature below its melting point.

$$\Delta_{sub} H = \Delta_{fus} H + \Delta_{vap} H$$

Metals have high enthalpy of sublimation.

49. Analysis of NH_4^+ cation

Take small amount of salt in the test tube and add NaOH and heat. Add Nessler's reagent $[K_2H_gI_4]$ to the solution, brown ppt is obtained. It confirmed NH_4^+ ion.



$+ 4KI + 3NaI + 2H_2O$

50. Permanganate ion can oxidise maganeous ion. The permanganate is also an oxidising agent. Stoichiometrically, 1.92 mg/L of potassium permanganate will further oxidise 1 mg/L of manganeous ion.

Mathematics

1. Given, $2|x|^2 + 51 = |1 + 20x|$

Then, following two possible cases arise:

***Case* I** When $1 + 20x > 0 \Rightarrow x > -\dfrac{1}{20}$

$\therefore$ $2x^2 + 51 = (1 + 20x)$

$\Rightarrow$ $2x^2 - 20x + 50 = 0$

$\Rightarrow$ $x^2 - 10x + 25 = 0$ $[\because 2 \neq 0]$

$\Rightarrow$ $(x - 5)^2 = 0$

$\Rightarrow$ $x = 5, 5$

***Case* II** When $1 + 20x < 0 \Rightarrow x < -\dfrac{1}{20}$

$\therefore$ $2x^2 + 51 = -(1 + 20x)$

$\Rightarrow$ $2x^2 - 20x + 52 = 0$

$\Rightarrow$ $x^2 - 10x + 26 = 0$

Here, $D < 0$

Thus, roots are imaginary.

Hence, sum of real roots = 5 + 5 = 10

2. Given differential equation is

$$\frac{dy}{dx} = \frac{1}{x + y + 1}$$

or $\dfrac{dx}{dy} = x + y + 1 \Rightarrow \dfrac{dx}{dy} - x = y + 1$ ...(i)

Eq. (i) is of the type $\dfrac{dx}{dy} + Px = Q$, where P and Q are functions of y or constant terms.

Here, $P = -1$

and $Q = y + 1$

$\therefore$ $IF = e^{\int P dy} = e^{\int (-1)\, dy} = e^{-y}$

Now, general solution is given by

$$x \cdot IF = \int IF \cdot Q\, dy + C_1$$

$\Rightarrow$ $x \cdot e^{-y} = \int \underset{II}{e^{-y}} \underset{I}{(y + 1)}\, dy + C_1$

$\Rightarrow$ $x e^{-y} = (y + 1)\left(\dfrac{e^{-y}}{-1}\right) + \int 1 \cdot e^{-y}\, dy + C_1$

$\Rightarrow$ $x e^{-y} = -e^{-y}(y + 1) - e^{-y} + C_1$

$\Rightarrow$ $x = -(y + 1) - 1 + C_1 e^{y}$ [on dividing by e^{-y}]

$\Rightarrow$ $x + y + 2 = C_1 e^{y}$

On taking log both sides, we get

$\log(x + y + 2) = \log C_1 e^{y}$

$\Rightarrow$ $\log(x + y + 2) = \log C_1 + \log e^{y}$

$[\because \log mn = \log m + \log n]$

$\Rightarrow$ $\log(x + y + 2) = C + y$ [put $C = \log C_1$]

which is the required solution.

3. Given roots are $\dfrac{1}{3 + \sqrt{2}}$ and $\dfrac{1}{3 - \sqrt{2}}$

i.e. $\dfrac{1}{(3 + \sqrt{2})} \times \dfrac{(3 - \sqrt{2})}{(3 - \sqrt{2})}$ and $\dfrac{1}{(3 - \sqrt{2})} \times \dfrac{(3 + \sqrt{2})}{(3 + \sqrt{2})}$

$\Rightarrow$ $\dfrac{3 - \sqrt{2}}{7}$ and $\dfrac{3 + \sqrt{2}}{7}$

So, the required quadratic equation is

$x^2 -$ (Sum of the roots) x + Product of the roots = 0

$\Rightarrow x^2 - \left[\dfrac{3 - \sqrt{2}}{7} + \dfrac{3 + \sqrt{2}}{7}\right]x + \left(\dfrac{3 - \sqrt{2}}{7}\right)\left(\dfrac{3 + \sqrt{2}}{7}\right) = 0$

$\Rightarrow$ $x^2 - \left(\dfrac{6}{7}\right)x + \dfrac{9 - 2}{49} = 0$

$\Rightarrow$ $x^2 - \dfrac{6}{7}x + \dfrac{7}{49} = 0$

$\Rightarrow$ $7x^2 - 6x + 1 = 0$

4. Since, image of a point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

So, image of point (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is

$$\frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = \frac{-2(2 - 3 + 4 + 3)}{4 + 1 + 1}$$

$\Rightarrow$ $\dfrac{x - 1}{2} = \dfrac{y - 3}{-1} = \dfrac{z - 4}{1} = -2$

$\Rightarrow$ $x = +1 - 4,\ y = 3 + 2,\ z = -2 + 4$

$\Rightarrow$ $x = -3,\ y = 5,\ z = 2$

i.e. required point is (–3, 5, 2).

5. Gorden Moore

6. Vector which is perpendicular to vertors **a** and **b**, is **a** × **b** or **b** × **a**.

Hence, two vectors are possible which are perpendicular to **a** and **b**.

7. Given that,

$$I = \int_0^6 \frac{dx}{1 + x^2} \quad \text{...(i)}$$

From Eq. (i), $f(x) = \dfrac{1}{1 + x^2}$

Now, divide the interval [0, 6] into six parts each of width

$$h = \frac{6-0}{6} = 1$$

The value of $f(x)$ are given below

x	0	1	2	3	4	5	6
$f(X)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

The trapezoidal rule is

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})]$$

$$\therefore \int_0^6 \frac{1}{1+x^2}\,dx = \frac{1}{2}[(1 + 0.027) + 2\{0.5 + 0.2 + 0.1 + 0.0588 + 0.0385\}]$$

$$= \frac{1}{2}[1.027 + 2(0.8973)]$$

$$= \frac{1}{2}[1.027 + 1.7946]$$

$$= \frac{1}{2}[2.8216] = 1.4108$$

8. Given,

$$\frac{dr}{dt} = 2\text{ m/s} \quad \ldots(i)$$

$$\frac{dh}{dt} = -3\text{ m/s}$$

$\because$ Volume of cylinder, $V = \pi r^2 h$

$$\therefore \quad \frac{dV}{dt} = \pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$$

At $r = 3$ m and $h = 5$ m,

$$\frac{dV}{dt} = \pi[2 \times 3 \times 5 \times 2 - 9 \times 3] \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \frac{dV}{dt} = \pi[60 - 27] = 33\pi$$

$$\Rightarrow \quad \frac{dV}{dt} = 33\pi\ \text{m}^3/\text{s}$$

9. Let required point be (α, β) on the straight line $y = 2x + 11$, which is nearest to the circle

$$16(x^2 + y^2) + 32x - 8y - 50 = 0$$

$$\Rightarrow \quad x^2 + y^2 + 2x - \frac{1}{2}y - \frac{50}{16} = 0$$

$\therefore$ Centre of circle $= \left(-1, \frac{1}{4}\right)$

and radius, $r = \sqrt{1 + \frac{1}{16} + \frac{50}{16}} = \frac{\sqrt{67}}{4}$

Now, equation of straight line passing through centre $\left(-1, \frac{1}{4}\right)$ and (α, β) is

$$y - \frac{1}{4} = \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right)(x + 1) \quad \ldots(i)$$

Now, gradient of this straight line $= \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right)$

Since, straight line (i) is perpendicular to the line $y = 2x + 11$

$$\therefore \quad \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right) \times 2 = -1 \quad [\because m_1 \cdot m_2 = -1]$$

$$\Rightarrow \quad 2\beta - \frac{1}{2} = -\alpha - 1$$

$$\Rightarrow \quad 2\beta + \alpha = -1 + \frac{1}{2} = \frac{-1}{2}$$

$$\Rightarrow \quad 4\beta + 2\alpha = -1 \quad \ldots(ii)$$

$\because$ Point (α, β) lies on straight line $y = 2x + 11$

$$\therefore \quad \beta = 2\alpha + 11$$

$$\Rightarrow \quad \beta - 2\alpha = 11 \quad \ldots(iii)$$

On solving Eqs. (ii) and (iii), we get

$$5\beta = 10 \Rightarrow \beta = 2$$

and $2\alpha = 2 - 11 = -9 \Rightarrow \alpha = \frac{-9}{2}$

$\therefore$ Required point is $\left(\frac{-9}{2}, 2\right)$.

10. Given equation is

$$b^2x^2 + y^2 = a^2b^2$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{a^2b^2} = 1 \quad \ldots(i)$$

Above equation of an ellipse with semi-major axis (a) and semi-minor axis (ab).

Now, ecentricity,

$$e = 1 - \frac{a^2b^2}{a^2}$$

$$\Rightarrow \quad b^2 = 1 - e^2 \quad \ldots(ii)$$

Let (x, y) be extrimities of latusrectum, then

$$x = ae \text{ and } y = \pm\frac{a^2b^2}{a}$$

$$\Rightarrow \quad \frac{x}{a} = e \text{ and } \frac{y}{a} = \pm b^2$$

From Eq. (ii), we get

$$\pm\frac{y}{a} = 1 - \frac{x^2}{a^2} \Rightarrow \pm ay + x^2 = a^2$$

Hence, locus of latusrectum is $x^2 \pm ay = a^2$.

11. Given differential equation is

$$(y\log x - 1)\,y\,dx = x\,dy$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(y\log x - 1)y}{x}$$

$$= \frac{y^2\log x}{x} - \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \frac{y^2\log x}{x}$$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} + \frac{y^{-1}}{x} = \frac{\log x}{x} \quad \ldots(i)$$

Put $\quad y^{-1} = v \Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$

$$\Rightarrow \quad y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

From Eq. (i), we have

$$-\frac{dv}{dx} + \frac{v}{x} = \frac{\log x}{x}$$

$$\Rightarrow \quad \frac{dv}{dx} - \frac{v}{x} = -\frac{\log x}{x} \quad \ldots(ii)$$

This is linear differential equation.

Here, $\text{IF} = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\log x} = \frac{1}{x}$

So, solution is $v \cdot \text{IF} = \int \text{IF} \cdot Q\,dx + C$

$$v \cdot \frac{1}{x} = \int \frac{1}{x}\left(-\frac{\log x}{x}\right)dx + C$$

$$\Rightarrow \quad v \cdot \frac{1}{x} = -\int \frac{\log x}{x^2}\,dx + C$$

$$= -\left[\log x\left(\frac{-1}{x}\right) + \int \frac{1}{x} \cdot \frac{1}{x}\,dx\right] + C$$

$$\Rightarrow \quad \frac{1}{x \cdot y} = \frac{\log x}{x} + \frac{1}{x} + C \quad \left[\because v = \frac{1}{y}\right]$$

$$\Rightarrow \quad 1 = y[\log x + 1 + Cx]$$

$$\Rightarrow \quad 1 = y[\log x + \log e + Cx] \quad [\because 1 = \log e]$$

$$\Rightarrow \quad 1 = y[\log e \cdot x + Cx]$$

12. A particular thing is received by a man with probability, $p = \frac{a}{a+b}$ and by a woman with probability, $q = \frac{b}{a+b}$.

Now, this experiment is repeated m times. Since, probability in each trial remains same for the men or women. Thus, we can apply binomial distribution.

Since, men should get odd number of things, so the random variable x would have $x = 1, 3, 5, \ldots$

$\therefore$ Required probability

$$= P(X = 1) + P(X = 3) + P(X = 5) + \ldots$$

$$= {}^mC_1pq^{m-1} + {}^mC_3p^3q^{m-3} + {}^mC_5p^5q^{m-5} + \ldots$$

$$= \left[\frac{(q+p)^m - (q-p)^m}{2}\right]$$

$$= \left[\frac{1 - \left(\frac{b}{a+b} - \frac{a}{a+b}\right)^m}{2}\right] \quad [\because p + q = 1]$$

$$= \frac{(a+b)^m - (b-a)^m}{2(a+b)^m}$$

13. Given,

$$I = \int(\sqrt{\tan x} + \sqrt{\cot x})\,dx,\ x \in \left(0, \frac{\pi}{2}\right)$$

$$= \int\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$

$$= \frac{\sqrt{2}}{\sqrt{2}}\int\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\,dx$$

$$= \sqrt{2}\int\frac{\sin x + \cos x}{\sqrt{2\sin x \cos x}}\,dx \quad \ldots(i)$$

Let $\quad \sin x - \cos x = t \quad \ldots(ii)$

$$\Rightarrow \quad (\cos x + \sin x)\,dx = dt$$

From Eq. (ii), we have

$$(\sin x - \cos x)^2 = t^2$$

$$\Rightarrow \quad 1 - 2\sin x \cos x = t^2$$

$$\Rightarrow \quad 2\sin x \cos x = 1 - t^2$$

So, Eq. (i) becomes

$$I = \sqrt{2}\int\frac{dt}{\sqrt{1 - t^2}}$$

$$= \sqrt{2}\sin^{-1}(t) + C$$

$$\Rightarrow \quad I = \sqrt{2}\sin^{-1}(\sin x - \cos x) + C$$

14. Given differential equation is

$$\sqrt{a + x}\frac{dy}{dx} + xy = 0$$

$$\Rightarrow \quad \sqrt{a + x}\frac{dy}{dx} = -xy$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{-x}{\sqrt{a + x}}\,dx$$

On integrating both sides, we get

$$\log y - \log C = \int\left[\frac{-x - a}{\sqrt{a + x}} + \frac{a}{\sqrt{a + x}}\right]dx$$

$$\Rightarrow \quad \log\left(\frac{y}{C}\right) = -\int\sqrt{x + a}\,dx + a\int\frac{1}{\sqrt{a + x}}\,dx$$

$$= -\frac{(x+a)^{3/2}}{3/2} + a\frac{(a+x)^{1/2}}{1/2}$$

$$\Rightarrow \log\left(\frac{y}{C}\right) = -\frac{2}{3}(x+a)\sqrt{x+a} + 2a\sqrt{x+a}$$

$$= \frac{2}{3}(2a-x)\sqrt{x+a}$$

$$\Rightarrow \frac{y}{C} = e^{\frac{2}{3}(2a-x)\sqrt{x+a}}$$

$$\Rightarrow y = C\cdot e^{\frac{2}{3}(2a-x)\sqrt{x+a}}$$

which is the required solution.

15. Given function is

$$f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$$

For increasing function, $f'(x) > 0, \forall x$

$\Rightarrow f'(x) = 2e^x + ae^{-x} + (2a+1) > 0$

$$\Rightarrow (2e^x + 1) + a(e^{-x} + 2) > 0$$

$$\Rightarrow a(e^{-x} + 2) > -(1 + 2e^x)$$

$$\Rightarrow a > \frac{-(1+2e^x)}{(e^{-x}+2)}$$

$$\Rightarrow a > \frac{-(1+2e^x)}{(1+2e^x)}\cdot e^x$$

$$\Rightarrow a > -e^x$$

$\because$ $e^x \in (0, \infty) \Rightarrow -e^x \in (-\infty, 0)$

and $a > -e^x$

$\therefore$ $a \in [0, \infty)$

16. Given, $f(x) = x^4 - x - 10$

We assume $x_0 = 2$ is the approximate root of $f(x)$.

Then, $h = -\frac{f(x_0)}{f'(x_0)} = -\frac{f(2)}{f'(2)}$

$$\Rightarrow h = -\left[\frac{(2)^4 - 2 - 10}{4(2)^3 - 1}\right]$$

$$\Rightarrow h = -\left[\frac{16-12}{31}\right] = \frac{-4}{31}$$

$$\Rightarrow h = -0.129$$

$\therefore$ Positive square root of $f(x)$ by Newton-Raphson method,

$$x_1 = x_0 + h = 2 + (-0.129)$$
$$= 2 - 0.129 = 1871$$

17. Given, $\mathbf{a} = \frac{\hat{\mathbf{i}} - 2\hat{\mathbf{j}}}{\sqrt{5}}$ and $\mathbf{b} = \frac{2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{14}}$

Now, $(2\mathbf{a} + \mathbf{b})\cdot[(\mathbf{a}\times\mathbf{b})\times(\mathbf{a} - 2\mathbf{b})]$

$= (2\mathbf{a} + \mathbf{b})\cdot[(\mathbf{a}\times\mathbf{b})\times\mathbf{a} - 2(\mathbf{a}\times\mathbf{b})\times\mathbf{b}]$

$= (2\mathbf{a} + \mathbf{b})\cdot[(\mathbf{a}\cdot\mathbf{a})\mathbf{b} - (\mathbf{b}\cdot\mathbf{a})\mathbf{a} - 2\{(\mathbf{a}\cdot\mathbf{b})\mathbf{b} - (\mathbf{b}\cdot\mathbf{b})\mathbf{a}\}]$

$= (2\mathbf{a} + \mathbf{b})[|\mathbf{a}|^2\mathbf{b} - 0\cdot\mathbf{a} - 2\{0\cdot\mathbf{b} - |\mathbf{b}|^2\mathbf{a}\}]$ $[\because \mathbf{a}\cdot\mathbf{b} = 0]$

$= (2\mathbf{a} + \mathbf{b})[1\cdot\mathbf{b} + 2\cdot 1\cdot\mathbf{a}]$ $[\because |\mathbf{a}| = 1, |\mathbf{b}| = 1]$

$= (2\mathbf{a} + \mathbf{b})(2\mathbf{a} + \mathbf{b})$

$= 4|\mathbf{a}|^2 + |\mathbf{b}|^2 + 4\mathbf{a}\cdot\mathbf{b}$

$= 4\times 1 + 1 + 4\times 0$ $[\because |\mathbf{a}| = 1 = |\mathbf{b}|$ and $\mathbf{a}\cdot\mathbf{b} = 0]$

$= 4 + 1 = 5$

18. Given, $3a = b + c$

$\because$ $\cot\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

and $\cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$\therefore \cot\left(\frac{B}{2}\right)\cdot\cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} \times \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \frac{s}{(s-a)} = \frac{2s}{2s-2a}$$

On putting $2s = a + b + c$, we get

$$\cot\left(\frac{B}{2}\right)\cdot\cot\left(\frac{C}{2}\right) = \frac{a+b+c}{a+b+c-2a}$$

$$\Rightarrow \cot\left(\frac{B}{2}\right)\cdot\cot\left(\frac{C}{2}\right) = \frac{a+3a}{a+3a-2a} = \frac{4a}{2a} = 2$$

19. Given, $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \sin^{-1}x = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\left[\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right]$$

$$\Rightarrow x = \sin\left\{\tan^{-1}\left(\frac{1}{2}\right)\right\}$$

$$\Rightarrow x = \sin\left\{\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right\}$$

$$\Rightarrow x = \frac{1}{\sqrt{5}}$$

20. Given, $I = \int_0^{\sqrt{\ln(\pi/2)}} \cos(e^{x^2})\cdot 2xe^{x^2}\,dx$

Put $e^{x^2} = t \Rightarrow 2xe^{x^2}\,dx = dt$

Now, lower limit, $t = 1$

Upper limit, $t = e^{\ln \pi/2} = \frac{\pi}{2}$

$\therefore$ $I = \int_1^{\pi/2}\cos(t)\,dt = [\sin t]_1^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin 1$

$\Rightarrow$ $I = 1 - \sin 1$

21. Application Programming Interface.

22. Given that,

$$P(A) = \frac{1}{4}$$

$$\Rightarrow P(A^C) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(A\cup B) = \frac{1}{2}$$

$$\therefore P\left(\frac{B}{A^C}\right) = \frac{P(B\cap A^C)}{P(A^C)}$$

$$= \frac{P(B) - P(A\cap B)}{P(A^C)}$$

$$= \frac{P(B) - [P(A) + P(B) - P(A\cup B)]}{P(A^C)}$$

$$= \frac{P(B) - P(A) - P(B) + P(A\cup B)]}{P(A^C)}$$

$$= \frac{-P(A) + P(A\cup B)}{P(A^C)}$$

$$= \frac{-\frac{1}{4} + \frac{1}{2}}{\frac{3}{4}} = \frac{-1+2}{\frac{3}{4}} = \frac{1}{3}$$

23. The distance of a point $P(x_1, y_1, z_1)$ from a plane $ax + by + cz + d = 0$ is

$$D = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

$\therefore$ Distance of the point (2, 3, 4) from the plane $3x - 6y + 2z + 11 = 0$ is

$$D = \left|\frac{6 - 18 + 8 + 11}{\sqrt{9 + 36 + 4}}\right|$$

$$= \left|\frac{7}{\sqrt{49}}\right| = \left|\frac{7}{7}\right|$$

$\Rightarrow$ $D = 1$ unit

24. Given, $Z = i\log(2 - \sqrt{3})$

$\Rightarrow \cos Z = \cos[i\log(2 - \sqrt{3})]$

$= \cos h\{\log(2 - \sqrt{3})\}$ $[\because \cos ix = \cosh x$, for $x \in R]$

$$= \frac{e^{\log(2-\sqrt{3})} + e^{-\log(2-\sqrt{3})}}{2} \quad \left[\because \cos hx = \frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{e^{\log(2-\sqrt{3})} + e^{\log\left(\frac{1}{2-\sqrt{3}}\right)}}{2}$$

$$= \frac{2 - \sqrt{3} + \frac{1}{2-\sqrt{3}}}{2}$$

$$= \frac{(2-\sqrt{3})^2 + 1}{2(2-\sqrt{3})} = \frac{4 + 3 - 4\sqrt{3} + 1}{2(2-\sqrt{3})}$$

$$= \frac{8 - 4\sqrt{3}}{2(2-\sqrt{3})} = \frac{4(2-\sqrt{3})}{2(2-\sqrt{3})} = 2$$

25. Given,

$$2x - \begin{bmatrix}1 & 2\\ 7 & 4\end{bmatrix} = \begin{bmatrix}3 & 2\\ 0 & -2\end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix}1 & 2\\ 7 & 4\end{bmatrix} + \begin{bmatrix}3 & 2\\ 0 & -2\end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix}1+3 & 2+2\\ 7+0 & 4-2\end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix}4 & 4\\ 7 & 2\end{bmatrix}$$

$$\Rightarrow x = \frac{1}{2}\begin{bmatrix}4 & 4\\ 7 & 2\end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix}2 & 2\\ 7/2 & 1\end{bmatrix}$$

26. Given, $\lim_{x\to 2}\frac{2 - \sqrt{2+x}}{2^{1/3} - (4-x)^{1/3}}$ $\left[\frac{0}{0}\text{form}\right]$

$$= \lim_{x\to 2}\frac{0 - \frac{1}{2\sqrt{2+x}}}{0 - \frac{1}{3}\cdot\frac{(-1)}{(4-x)^{2/3}}}$$

$$= \lim_{x\to 2}\frac{-3(4-x)^{2/3}}{2\sqrt{2+x}}$$

$$= \frac{-3}{2}\times\frac{(4-2)^{2/3}}{\sqrt{2+2}}$$

$$= \frac{-3}{2}\times\frac{2^{2/3}}{2} = -3\cdot 2^{\frac{2}{3}-2}$$

$$= -3\cdot 2^{-4/3}$$

27. Given lines of regressions are

$$x + 2y - 5 = 0$$

and $2x + ky - 8 = 0$

Rewriting above equations,

$$y = -\frac{1}{2}x + \frac{5}{2} \text{ and } x = -\frac{k}{2}y + 4$$

Hence, $b_{yx}(r_1) = \frac{-1}{2}$ and $b_{xy}(r_2) = \frac{-k}{2}$

$\because \quad r^2 = b_{yx} \times b_{xy}$

$$r^2 = -\frac{1}{2} \times -\frac{k}{2}$$

$$r^2 = \frac{k}{4} \Rightarrow r = \frac{\sqrt{k}}{2}$$

Given, $\sigma_x^2 = 12$

$\Rightarrow \quad \sigma_x = \sqrt{12}$

$\sigma_y^2 = 4$

$\Rightarrow \quad \sigma_y = 2$

We know that,

$$b_{xy} = \frac{r\sigma_x}{\sigma_y}$$

$$\therefore \quad \frac{-k}{2} = \frac{\sqrt{k} \times \sqrt{12}}{2\sqrt{2}}$$

$\Rightarrow \quad -2k = \sqrt{12k}$

$\Rightarrow \quad 4k^2 = 12k$

$\Rightarrow \quad k = 3$

28. Three lines of triangle are given by

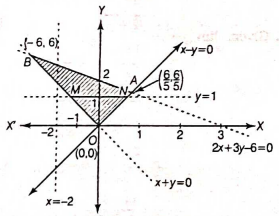
$$(x^2 - y^2)(2x + 3y - 6) = 0$$

$\Rightarrow \quad (x - y)(x + y)(2x + 3y - 6) = 0$

$\therefore$ The three lines of triangle are

$$x - y = 0, \; x + y = 0$$

and $\quad 2x + 3y - 6 = 0$



From given lines of triangle, the required ΔOAB is formed.

$\because (-2, \lambda)$ lies inside the triangle.

$\therefore \quad 2(-2) + 3(\lambda) - 6 < 0$ and $-2 + \lambda > 0$

$\Rightarrow \quad -4 + 3\lambda - 6 < 0$ and $\lambda > 2$

$\Rightarrow \quad 3\lambda < 10$ and $\lambda > 2$

$\Rightarrow \quad \lambda < \frac{10}{3}$ and $\lambda > 2$

$$\therefore \quad \lambda \in \left(2, \frac{10}{3}\right) \quad \ldots(i)$$

Now, $(\mu, 1)$ lies outside the triangle.

To find value of μ, we find the interval $[M, N]$ for values of x.

$x + 1 \geq 0$ and $x - 1 \leq 0$

$\Rightarrow \quad x \geq -1$ and $x \leq 1$

$\therefore \quad x \in [-1, 1]$

$\because (\mu, 1)$ lies outside the triangle.

$\therefore \quad \mu \in (-\infty, -1) \cup (1, \infty)$

or $\quad \mu \in R - [-1, 1]$

29. System software : Utility software :: Operating system : Arti-virus

Utility software-Anti-virus

30. Given, $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin[x]}{[x]}$$

$$= \lim_{h \to 0} \frac{\sin[0 + h]}{[0 + h]} = 1 \quad \ldots(i)$$

[from RHS of zero, $[0 + h] = 0$]

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin[x]}{[x]}$$

$$= \lim_{h \to 0} \frac{\sin[0 - h]}{[0 - h]} = \frac{\sin(-1)}{(-1)}$$

[from LHS of zero, $[0 - h] = -1$]

$= \sin(1) \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

LHL $\neq$ RHL

$\therefore \lim_{x \to 0} f(x)$ does not exist.

31. Given equations of circles are

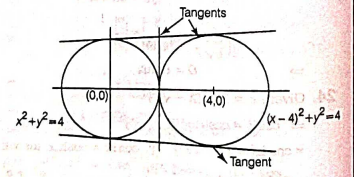
$$x^2 + y^2 = 4$$

and $\quad x^2 + y^2 - 8x + 12 = 0$

or $\quad x^2 + y^2 = (2)^2 \quad \ldots(i)$

$$(x - 4)^2 + (y - 0)^2 = (2)^2 \quad \ldots(ii)$$

The figure of both the circles are shown below :



From figure, we see that there is exactly three common tangents.

32. Given, $f(x) = \int_2^x \frac{dt}{\sqrt{1 + t^4}}$

Using Leibnitz's rule, we have

$$\frac{d}{dx} f(x) = f'(x) = \frac{1}{\sqrt{1 + x^4}} \frac{d}{dx}(x) - 0$$

$$\Rightarrow \quad f'(x) = \frac{1}{\sqrt{1 + x^4}}$$

It is given that, g is inverse of f.

i.e. $\quad f^{-1} = g$

Then, $\quad (gof)(x) = x \quad [\because f^{-1} = g]$

On differentiating, we have

$g'(f(x)) \cdot f'(x) = 1 \quad$ [by chain rule]

$\Rightarrow \quad g'(f(x)) = \frac{1}{f'(x)}$

$\because \quad f(x) = 0$, if $x = 2$

$\therefore \quad g'(f(2)) = \frac{1}{f'(2)}$

$\Rightarrow \quad g'(0) = \frac{1}{f'(2)} = \frac{1}{\frac{1}{\sqrt{1 + 2^4}}}$

$\Rightarrow \quad g'(0) = \sqrt{17}$

33. Let $f(x) = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx$

Then, $f(x)$ is a polynomial.

So, it is continuous in R.

Now, $\quad f(0) = 0$

and $\quad f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a + 3b + 6c}{6}$

$\Rightarrow \quad f(1) = 0$

$[\because 2a + 3b + 6c = 0$, given$]$

$\because f(x)$ is a polynomial, so it is differentiable in R, so in $(0, 1)$.

Hence, by Rolle's theorem, there exists atleast one point $x \in (0, 1)$, there exists such that

$$f'(x) = 0$$

$\Rightarrow \quad ax^2 + bx + c = 0$

Hence, required interval is $(0, 1)$.

34. Given, for $x, y \in N$,

$$f(x + y) = f(x) \cdot f(y)$$

Then, function will be of the form

$f(x) = a^x$, where $a \in N \quad [\because a \neq 1]$

$\because \quad f(1) = 3$

$\Rightarrow \quad f(1) = a^1 = 3$

$\Rightarrow \quad a = 3$

$\therefore$ Function is $f(x) = 3^x$.

Now, $\quad \sum_{x=1}^{n} f(x) = 120$

$\Rightarrow \quad \sum_{x=1}^{n} 3^x = 120$

$\Rightarrow \quad 3 + 3^2 + 3^3 + \ldots + 3^n = 120$

$\Rightarrow \quad \frac{3(3^n - 1)}{3 - 1} = 120$

$\Rightarrow \quad 3^n = 1 + \frac{120 \times 2}{3}$

$\Rightarrow \quad 3^n = 81 = 3^4$

So, $\quad n = 4$

35. $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

$= \hat{i} \cdot (\hat{i}) + \hat{j} \cdot (\hat{j}) + \hat{k} \cdot (\hat{k}) \quad [\because \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}, \hat{i} \times \hat{j} = \hat{k}]$

$= 1 + 1 + 1 = 3 \quad [\because \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1]$

36. Given function is

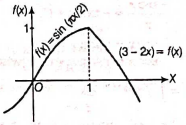
$$f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right), & \text{if } x < 1 \\ 3 - 2x, & \text{if } x \leq 1 \end{cases}$$

$\because$ LHL and RHL of $f(x)$ is 1, when $x \to 1$

and $\quad f(1) = 3 - 2 = 1$

$\therefore \quad f(x)$ is continuous at $x = 1$

Now, graph of function $f(x)$ is



From the above graph of function, we see that $f(x)$ has local maxima at $x = 1$.

37. It is given that, $\sin\alpha$ and $\cos\alpha$ are the roots of $ax^2 + bx + c = 0$.

$$\therefore \quad \text{Sum of the roots} = \sin\alpha + \cos\alpha = \frac{-b}{a}$$

and product of the roots $= \sin\alpha \cdot \cos\alpha = \frac{c}{a}$

$\because \quad (\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$

$\therefore \quad \left(\frac{-b}{a}\right)^2 = 1 + 2\left(\frac{c}{a}\right) \quad [\because \sin^2\alpha + \cos^2\alpha = 1]$

$\Rightarrow \quad \frac{b^2}{a^2} = \frac{a + 2c}{a}$

$\Rightarrow \quad b^2 = a(a + 2c)$

$\Rightarrow \quad b^2 = a^2 + 2ac$

$\Rightarrow \quad a^2 - b^2 + 2ac = 0$

38. Given differential equation is

$$\frac{dy}{dx} = y\tan x - y^2 \sec x$$

$$\Rightarrow \quad \frac{dy}{dx} - y\tan x = -y^2 \sec x$$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} - \frac{\tan x}{y} = -\sec x \quad \text{...(i)}$$

Put $\quad \frac{-1}{y} = u$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} = \frac{du}{dx}$$

From Eq. (i),

$$\frac{du}{dx} + \tan x \cdot u = -\sec x \quad \text{...(ii)}$$

This is linear differential equation of the form

$\frac{du}{dx} + P \cdot u = Q$, where $P = \tan x, Q = -\sec x$

$$\therefore \quad \text{IF} = e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$$

Hence, general solution is

$$u \cdot \text{IF} = \int \text{IF} \cdot Q\, dx + C_1$$

$$\Rightarrow \quad u \cdot \sec x = \int (\sec x) \cdot (-\sec x)\, dx + C_1$$

$$\Rightarrow \quad u \cdot \sec x = -\int \sec^2 x\, dx + C_1$$

$$\Rightarrow \quad u \sec x = -\tan x + C_1$$

$$\Rightarrow \quad \frac{-\sec x}{y} = -\tan x + C_1 \quad \left[\because u = \frac{-1}{y}\right]$$

$$\Rightarrow \quad \sec x = y(\tan x - C_1)$$

$$\Rightarrow \quad \sec x = y(\tan x + C) \quad [\text{where, } C = -C_1]$$

39. Given, $I = \int_0^{100} e^{x - [x]}\, dx$

$\because x - [x] = \{x\}$, where $\{\}$ is fractional part function.

$$\therefore \quad I = \int_0^{100} e^{\{x\}}\, dx$$

$\because \{x\}$ is periodic function with period 1.

$$\therefore \quad I = 100\int_0^1 e^{\{x\}}\, dx = 100\int_0^1 e^x\, dx$$

$[\because \{x\} = x, \text{ if } x \in [0, 1)]$

$$= 100[e^x]_0^1$$

$$= 100[e^1 - e^0] = 100(e - 1)$$

40. Given,

$$(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(\hat{i}x + \hat{j}y + \hat{k}z)$$

On equating the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ both sides, we have

$$x + 3y - 4z = \lambda x,$$

$$x - 3y + 5z = \lambda y$$

and $\quad 3x + y + 0 = \lambda z$

Above three equations can be rewritten as

$$(1 - \lambda)x + 3y - 4z = 0$$

$$x - (3 + \lambda)y + 5z = 0$$

$$3x + y - \lambda z = 0$$

This is homogeneous system of equations in three variables x, y and z.

It is consistent and have non-zero solution.

i.e. $(x, y, z) \neq (0, 0, 0)$, if determinant of coefficient matrix is zero.

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

On expanding along first row, we have

$$(1 - \lambda)[\lambda(3 + \lambda) - 5] - 3(-\lambda - 15) - 4(1 + 9 + 3\lambda) = 0$$

$$\Rightarrow (1 - \lambda)(\lambda^2 + 3\lambda - 5) + 3\lambda + 45 - 40 - 12\lambda = 0$$

$$\Rightarrow \quad \lambda^2 + 3\lambda - 5 - \lambda^3 - 3\lambda^2 + 5\lambda - 9\lambda + 5 = 0$$

$$\Rightarrow \quad -\lambda^3 - 2\lambda^2 - \lambda = 0$$

$$\Rightarrow \quad \lambda(\lambda^2 + 2\lambda + 1) = 0$$

$$\Rightarrow \quad \lambda(\lambda + 1)^2 = 0$$

$$\Rightarrow \quad \lambda = 0, -1$$

41. It is given, $\cot A$, $\cot B$ and $\cot C$ are in AP.

$$\Rightarrow \quad 2\cot B = \cot A + \cot C$$

$$\Rightarrow \quad 2\frac{\cos B}{\sin B} = \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C}$$

$$\because \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$$\Rightarrow \quad \frac{2\cos B}{kb} = \frac{\cos A}{ak} + \frac{\cos C}{ck}$$

$$\Rightarrow \quad \frac{2\cos B}{b} = \frac{\cos A}{a} + \frac{\cos C}{c}$$

$$\Rightarrow \quad \frac{2}{b}\left[\frac{a^2 + c^2 - b^2}{2ac}\right] = \frac{1}{a}\left[\frac{b^2 + c^2 - a^2}{2bc}\right] + \frac{1}{c}\left[\frac{a^2 + b^2 - c^2}{2ab}\right]$$

$$\Rightarrow \quad \frac{2(a^2 + c^2 - b^2)}{2abc} = \frac{1}{2abc}[(b^2 + c^2 - a^2) + (a^2 + b^2 - c^2)]$$

$$\Rightarrow \quad 2(a^2 + c^2 - b^2) = 2b^2$$

$$\Rightarrow \quad a^2 + c^2 = 2b^2$$

$\therefore a^2, b^2$ and c^2 are in AP.

42. Given integral is $\int_1^2 e^{-x/2}\, dx$

On dividing interval [1, 2] in four parts, we have

$$h = \frac{2 - 1}{4} = \frac{1}{4}$$

Now, value of $f(x) = e^{-x/2}$ is given below

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	e
$f(x)$	0.6065	0.5352	0.4724	0.4168	0.3679

Now, Simpson's $\frac{1}{3}$ rule is

$$\int_{x_0}^{x_0 + nh} f(x)\, dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})]$$

$$\therefore \int_1^2 e^{-x/2}\, dx = \frac{1}{12}[(0.6065 + 0.3679) + 4(0.5352 + 0.4168) + 2(0.4724)]$$

$$= \frac{1}{12}[0.9744 + 3.808 + 0.9448]$$

$$= \frac{1}{12} \times 5.7272 = 0.477$$

43. When a die is rolled three times.

Then, cardinality of sample space $= 6^3$

Now, according to question, the favourable outcomes

$= \{(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 3, 4), (1, 3, 5), (1, 3, 6), (1, 4, 5), (1, 4, 6), (1, 5, 6), (2, 3, 4), (2, 3, 5), (2, 3, 6), (2, 4, 5), (2, 4, 6), (3, 4, 5), (3, 4, 6), (4, 5, 6), (2, 5, 6), (3, 5, 6)\}$

$\therefore$ Required probability $= \frac{20}{6^3} = \frac{5}{54}$

44. Let the plane meets the axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$. Then, equation of plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

It is given that plane is at a constant distance p from the origin (0, 0, 0).

$$\therefore \quad p = \left|\frac{0 + 0 - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right|$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \quad \text{...(i)}$$

Let (α, β, γ) be the coordinates of centroid of the formed tetrahedron.

Then, $\quad \alpha = \frac{a + 0 + 0 + 0}{4} \Rightarrow a = 4\alpha$

$$\beta = \frac{0 + b + 0 + 0}{4} \Rightarrow b = 4\beta$$

$$\gamma = \frac{0 + 0 + c + 0}{4} \Rightarrow c = 4\gamma$$

On putting values of a, b, c in Eq. (i), we get

$$\frac{1}{p^2} = \frac{1}{(4\alpha)^2} + \frac{1}{(4\beta)^2} + \frac{1}{(4\gamma)^2}$$

$$\Rightarrow \quad \frac{16}{p^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$$

$\therefore$ Locus of centroid of tetrahedron is

$$\frac{16}{p^2} = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$$

45. Given, $f(x) = \log_e(6 - |x^2 + x - 6|)$

The function $f(x)$ is defined, if

$$(6 - |x^2 + x - 6|) > 0$$

$$\Rightarrow \quad |x^2 + x - 6| < 6$$

$$\Rightarrow \quad -6 < x^2 + x - 6 < 6$$

If $\quad x^2 + x - 6 < 6$

$$\Rightarrow \quad x^2 + x - 12 < 0$$

$$\Rightarrow \quad (x + 4)(x - 3) < 0$$

$+ \quad -4 \quad - \quad 3 \quad +$

$$\therefore \quad x \in (-4, 3) \quad \text{...(i)}$$

Now, if $-6 < x^2 + x - 6$

$$\Rightarrow \quad x^2 + x > 0 \Rightarrow x(x + 1) > 0$$

$+ \quad -1 \quad - \quad 0 \quad +$

$$\therefore \quad x \in (0, \infty) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$x \in (0, 3)$$

$\because f(x)$ has only two integral values.

$$\therefore \quad x = 1, 2$$

46. The given equation is

$$y^2 + z^2 = 0$$

Above equation represents a point circle in the YZ-plane.

47. Given complex numbers are

$$Z_1 = (1 + i) \equiv (1, 1) \equiv (a_1, b_1)$$

$$Z_2 = (-2 + 3i) \equiv (-2, 3) \equiv (a_2, b_2)$$

and $\quad Z_3 = \frac{ai}{3} \equiv \left(0, \frac{a}{3}\right) \equiv (a_3, b_3)$

These three points will be collinear, if

$$\frac{1}{2}[a_1(b_2 - b_3) + a_2(b_3 - b_1) + a_3(b_1 - b_2)] = 0$$

$$\Rightarrow \quad 1\left(3 - \frac{a}{3}\right) - 2\left(\frac{a}{3} - 1\right) + 0(1 - 3) = 0$$

$$\Rightarrow \quad \frac{9 - a}{3} - 2\left(\frac{a - 3}{3}\right) + 0 = 0$$

$$\Rightarrow \quad 9 - a - 2a + 6 = 0 \Rightarrow 3a = 15 \Rightarrow a = 5$$

48. In word BHARAT,
A appears 2 times
B, H, R, T appear 1 time.
If there is no restriction on the letters of the word BHARAT, then total number of different words formed $= \frac{6!}{2!}$

Now, take B and H always together, then total number of different words formed $= \frac{5!}{2!}$

$\therefore$ Total number of different words formed, when B and H are never together.

$$= \frac{6!}{2!} - \frac{5!}{2!} = \frac{1}{2!}(6! - 5!)$$

$$= \frac{5!(6-1)}{2} = \frac{120 \times 5}{2} = 300$$

49. Given that, a, b and c are in HP.

$\therefore$ Harmonic mean of a and c is b.

and geometric mean of a and c is $\sqrt{ac}$.

$\because$ Geometric mean > Harmonic mean

$$\Rightarrow \quad \sqrt{ac} > b \quad \ldots(i)$$

Now, for the positive numbers a^n and c^n, we have

Geometric mean $= \sqrt{a^n \cdot c^n}$

and arithmetic mean $= \frac{a^n + c^n}{2}$

$\because$ AM > GM

$$\therefore \quad \frac{a^n + c^n}{2} > \sqrt{a^n \cdot c^n}$$

$$\Rightarrow \quad a^n + c^n > 2\sqrt{a^n \cdot c^n}$$

$$\Rightarrow \quad a^n + c^n > 2(\sqrt{ac})^n > 2b^n \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad a^n + c^n > 2b^n$$

50. $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right]$

$$= \tan\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1 - \frac{1}{25}}\right) - \frac{\pi}{4}\right]$$

$$\left[\because 2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1-\theta^2}\right)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{10}{24}\right) - \frac{\pi}{4}\right]$$

$$= \frac{\tan\left\{\tan^{-1}\left(\frac{10}{24}\right)\right\} - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left\{\tan^{-1}\left(\frac{10}{24}\right)\right\} \cdot \tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{\frac{10}{24} - 1}{1 + \frac{10}{24} \cdot 1} = \frac{10 - 24}{24 + 10}$$

$$= \frac{-14}{34} = \frac{-7}{17}$$