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CGPET2014 PAPER

WITH SOLUTION

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Engineering Entrance Exam

Physics

1. The work done in increasing the size of a rectangular soap film with dimensions $8 \text{ cm} \times 3.75 \text{ cm}$ to $10 \text{ cm} \times 6 \text{ cm}$ is 2×10^{-4} J. The surface tension of the film in N m^{-1} is

(a) 3.3×10^{-2} (b) 1.65×10^{-2}

(c) 8.25×10^{-2} (d) 6.6×10^{-2}

2. A body of mass m rises to a height $h = \frac{R}{5}$ from the earth's surface, where R is the earth's radius. If g is acceleration due to gravity at the earth's surface, the increase in potential energy will be

(a) mgh (b) $\frac{4}{5} mgh$ (c) $\frac{5}{6} mgh$ (d) $\frac{6}{7} mgh$

3. Two springs of spring constant 2000 Nm^{-1} and 1000 Nm^{-1} are stretched with same force. They will have potential energy in the ratio of

(a) 4 : 1 (b) 2 : 1

(c) 1 : 2 (d) 1 : 4

4. The energy required to break the covalent bond in a semiconductor is

(a) always 1 eV

(b) equal to the forbidden energy gap of semiconductor

(c) equal to fermi energy

(d) much less than fermi energy

5. Consider the following statements.

I. Random and excess exposure to X-rays may induce diseases.

II. X-rays has a damaging effect on the living cells of a body which may lead to cell death.

(a) Both I and II are true

(b) I is true but II is not true

(c) II is true but I is not true

(d) Both I and II are not true

6. In the potentiometer experiment, if deflection in galvanometer is measured zero, then the current will become zero in

(a) potentiometer wire

(b) galvanometer circuit

(c) main circuit

(d) cell

7. When a glass prism of refracting angle 60° is immersed in a liquid its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is

(a) 45° (b) 42°

(c) 50° (d) 52°

8. If one penetrates a uniformly charged conducting spherical shell, the electric field E is

(a) increases

(b) decreases

(c) remains same as it is on surface

(d) zero at all points

9. Faraday constant

(a) depends on the amount of the electrolyte

(b) depends on the current in the electrolyte

(c) is a universal constant

(d) depends on the amount of charge passed through the electrolyte

10. Interference effect is observed in

(a) only transverse wave

(b) only longitudinal wave

(c) Both (a) and (b)

(d) None of the above

11. A comb is run through wet hair on a rainy day, then

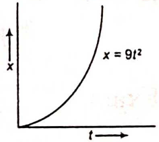
(a) it will attract large number of small bits of paper

(b) it will not go through the hair

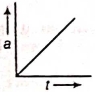
(c) it will not attract small bits of paper

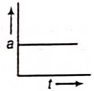
(d) None of the above

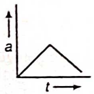
12. The displacement-time graph of a particle moving along a straight line is shown in the figure.

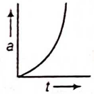


The acceleration-time graph of this particle is

(a) 

(b) 

(c) 

(d) 

13. Calculate the heat required to increase the temperature of 1 mole of one atomic gas from 0°C to 150°C, when no work is done. [$C_p = 2.5\,R$ and $R = 8.3$ J mol^{-1} K^{-1}]
(a) 867.5 J
(b) 186.5 J
(c) 1867.5 J
(d) 86.7 J

14. The maximum kinetic energy of photoelectrons coming out of a metal surface is 10 eV. The minimum voltage required to stop, the emission of electrons from this metal surface is
(a) 10 V (b) 5 V (c) –5 V (d) –10 V

15. Tangent galvanometer is not useful to measure current, because
(a) this is not directly readable
(b) same current will give different readings at different places
(c) near by magnetic material will effect the readings
(d) All of the above

16. Thermal radiation exist in which part of electromagnetic spectrum?
(a) Ultraviolet
(b) Infrared
(c) Visible
(d) Violet

17. Two similar cells are connected first in series and then in parallel, the ratio of balancing length on the potentiometer wire will be
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

18. The dimensions of $\frac{a}{b}$ in the equation $p = \frac{a - t^2}{bx}$, where p is pressure, x is distance and t is time, is
(a) $[M^2LT^{-3}]$ (b) $[MT^{-2}]$
(c) $[ML^3T^{-1}]$ (d) $[M^2L^{-3}T^2]$

19. A rod of length L is composed of a uniform length $\frac{L}{2}$ of wood whose mass in m_w and a uniform length $\frac{L}{2}$ of brass whose mass is m_b. The moment of inertia I of the rod about an axis perpendicular to the rod and through its centre is equal to
(a) $(m_w + m_b)\frac{L^2}{6}$ (b) $(m_w + m_b)\frac{L^2}{2}$
(c) $(m_w + m_b)\frac{L^2}{12}$ (d) $(m_w + m_b)\frac{L^2}{3}$

20. A particle, doing simple harmonic motion, at a distance 3 cm from mean position has acceleration 12 cm/s^2. What is its time period?
(a) 0.5 s (b) 1s (c) 2s (d) 3.14 s

21. If the earth did not rotate on its axis, the magnitude of the gravitational acceleration at the equator would be about
(a) 0.003 % larger (b) 0.3 % larger
(c) 0.3 % smaller (d) 0.003 % smaller

22. A charge particle enters a magnetic field B with its initial velocity v making an angle of 45° with B. The path of the charge particle will be
(a) a straight line (b) a circle
(c) an ellipse (d) a helix

23. A step up transformer has turn ratio 10 : 1. A cell of emf 2 V is fed to the primary, then the secondary voltage developed is
(a) 20 V (b) 10 V
(c) 2 V (d) zero

24. When yellow light is refracted by a prism in minimum deviation state, then
(a) angle of incidence is equal to angle of refraction
(b) angle of incidence is greater than the angle of refraction
(c) angle of incidence is less than angle of refraction
(d) sum of angle of incidence and angle of refraction is 90°

25. If in a Young double slit experiment maximum intensity is I_{max}, then intensity at $\lambda/2$ path difference, is
(a) I_{max} (b) $I_{max/2}$ (c) $I_{max/4}$ (d) zero

26. A man at a distance 11 km from two pillars wants to see two pillars separately. What will be the approximate distance between the pillars?
(a) 3 m (b) 1 m
(c) 0.25 m (d) 0.5 m

27. Dimensions of Stefan's constant is
(a) $[MLT^{-3}\theta^{-4}]$ (b) $[MT^{-3}\theta^{-4}]$
(c) $[M^2T^{-3}\theta^{-4}]$ (d) $[M^2T^{-2}\theta^{-4}]$

28. When a current changes from 2 A to 4 A in 0.05 s in a coil, induced emf is 8 V. The self-inductance of coil is
(a) 0.1 H (b) 0.2 H
(c) 0.4 H (d) 0.8 H

29. A force $F = Ay^2 + By + C$ acts on a body in the y-direction. Find the work done by this force during a displacement $y = -a$ to $y = +a$.
(a) $\frac{2Aa^3}{3}$
(b) $\frac{2Aa^3}{3} + 2Ca$
(c) $\frac{2Aa^3}{3} + \frac{Ba^3}{2} + Ca$
(d) None of these

30. Time period of oscillation of mass m suspended from a spring is T. What is the time period when the spring is cut in half and the same mass is suspended from one of the halves?
(a) $T/2$ (b) $T/\sqrt{2}$
(c) $\sqrt{2}\,T$ (d) $2\,T$

31. An electric current of 2 A passes through a wire of resistance 25 Ω. How much heat will be generated in 1 min?
(a) 6×10^3 J
(b) 3.6×10^3 J
(c) 0.6×10^3 J
(d) 0.36×10^3 J

32. Specific resistance of a conductor increases with
(a) increase in temperature
(b) increase in cross-section area
(c) increase in cross-section area and decrease in length
(d) decrease in cross-section area

33. For hydrogen like ions with z protons, the radius of nth orbit is given by r_n (where, a_0 is Bohr radius)
(a) $n^2 a_0 z^2$ (b) $\frac{n^2 a_0}{z}$
(c) $\frac{n^2 a_0}{z^2}$ (d) $\frac{n^3 a_0^2}{z^4}$

34. A mass m hanging from a spring is doing simple harmonic motion with frequency f. If the mass is increased by 4 times, then frequency will be
(a) $2\,f$ (b) $f/2$ (c) $4\,f$ (d) $f/4$

35. The SI unit of ε_0 in the formula of capacitance is given by
(a) microfarad/meter
(b) farad/meter
(c) meter^2/farad
(d) farad/centimeter

36. Distance between objective and eye-piece of a microscope is 20.6 cm. Consider both lens are thin and focal length of each lens is 6 mm. If last image is formed at infinity, then linear magnification of the objective is
(a) –1347 (b) –619
(c) –32.3 times (d) –3.23 times

37. Internal energy of a gas remains unchanged in
I. an isothermal process
II. an adiabatic process
III. a reversible process
IV. a cyclic process
Which of these are true?
(a) I and IV (b) I, III and IV
(c) III and IV (d) II and III

38. A projectile is thrown with an initial velocity of $\mathbf{u} = (a\,\hat{i} + b\,\hat{j})$ m/s. If the range of the projectile is double the maximum height reached by it, then
(a) $a = 2b$ (b) $b = 2a$
(c) $a = b$ (d) None of these

39. An electric charge in uniform motion produces
(a) only electric field
(b) only magnetic field
(c) Both electric and magnetic field
(d) Neither electric nor magnetic field

40. Time constant of a series R-C circuit is
(a) $+RC$ (b) $-RC$
(c) R/C (d) C/R

41. A magnet makes 25 oscillations in 5 min at one place, where as it takes 9 s to complete one oscillation at another place. The ratio of horizontal components of the earths magnetic field at these places $\frac{H_1}{H_2}=$

(a) $\frac{2}{7}$
(b) $\frac{81}{274}$
(c) $\frac{1}{8}$
(d) $\frac{9}{16}$

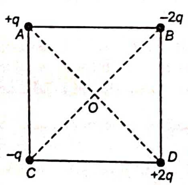
42. If λ is the incident wavelength and λ_0 is the threshold wavelength for a metal surface, photoelectric effect takes place only, if

(a) $\lambda \le \lambda_0$
(b) $\lambda \ge \lambda_0$
(c) $\lambda \ge 2\lambda_0$
(d) None of the above

43. The mass number of an atom is 15 and its atomic number is 7. Now, this atom absorbs an α-particle and emits a proton. What will be the mass number of changed atom?

(a) 16
(b) 18
(c) 17
(d) 15

44. What is the direction of the electric field at the centre O of the square in the figure shown below? Given that, $q = 10$ nC and the side of the square is 5 cm.



(a) at 45° to OA upward
(b) at 135° to OA towards BD
(c) no direction, because $E = 0$
(d) None of the above

45. Which equation is valid for adiabatic process?

(a) $TV^{\gamma-1}$ = constant
(b) $pV^{\gamma-1}$ = constant
(c) $T^{\gamma}V^{\gamma-1}$ = constant
(d) $\frac{p^{\gamma-1}}{T^{\gamma-1}}$ = constant

46. For changing the range of a galvanometer with G ohm resistance from V volt to n volt, what will be the value of resistance connected in series to it?

(a) $(n-1)G$
(b) G/n
(c) nG
(d) $\frac{G}{n-1}$

47. Which of the following relation correct? (v_{rms}-root mean square velocity, $\bar{v}$-mean velocity and v_{mp}-most probable velocity)

(a) $v_{rms} > \bar{v} < v_{mp}$
(b) $v_{rms} < \bar{v} > v_{mp}$
(c) $v_{rms} > \bar{v} > v_{mp}$
(d) None of the above

48. The effect of reverse bias in a junction diode on its potential barrier is

(a) increases
(b) decreases
(c) remains same
(d) None of the above

49. Which of the following is the Biot-Savart's law in vector form?

(a) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r}\hat{n}$
(b) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^3}\hat{n}$
(c) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}\hat{n}$
(d) None of the above

50. During an experiment, an ideal gas is found to obey an additional law Vp^2 = constant. The gas is initially at temperature T and volume V. The temperature of the gas will be following, when it expands to a volume $2V$?

(a) $\sqrt{2}\,T$
(b) $\sqrt{4}\,T$
(c) $\sqrt{6}\,T$
(d) $\sqrt{5}\,T$

Chemistry

1. Correct basicity of the following compounds are

I. Aniline II. Pyridine
III. Pyrrole IV. Guanidine

(a) I > II > III > IV (b) III > I > II > IV
(c) IV > II > I > III (d) II > IV > III > I

2. Lithium is the strongest reducing agent because of

(a) its greater hydration energy
(b) its high ionization energy
(c) its high electron affinity
(d) its low electronegativity

3. Which of the following is not basic amino acid?

(a) Leucine (b) Lysine
(c) Arginine (d) Histidine

4. Finely divided powder of charcoal adsorbs the substance X to a large extent. This is because

(a) the surface area of charcoal is increased
(b) charcoal powder can be spread over X homogeneously
(c) X can be spread over charcoal homogeneously
(d) charcoal is highly reactive

5. In qualitative analysis, NH_4Cl is added before NH_4OH

(a) to increase $[OH^-]$ concentration
(b) for making HCl
(c) to decrease $[OH^-]$ concentration
(d) statement is wrong

6. $C_2H_5Cl \xrightarrow[CuI]{Li} \xrightarrow{C_2H_5Cl} n$-butane

This method is

(a) Wurtz synthesis
(b) Kolbe synthesis
(c) Corey House synthesis
(d) Friedel-Craft synthesis

7. Certain electric current for half an hour can collect 11.2 L of hydrogen at NTP. Same current when passed through an electrolytic solution for one hour, can deposite how much silver?

(a) 216 g (b) 108 g (c) 47 g (d) 60 g

8. Which of the following complexes is an outer orbital complex?

(a) $[Co(NH_3)_6]^{3+}$ (b) $[Fe(CN)_6]^{4-}$
(c) $[Ni(NH_3)_6]^{2+}$ (d) $[Mn(CN)_6]^{4-}$

9. IUPAC name for the complex compound $K_3[Fe(CN_6)]$ is

(a) Potassium hexacyanoferrate [II]
(b) Potassium cynohexaferrate [II]
(c) Potassium hexacyanoferrate [III]
(d) Potassium ferrocyanide iron [III]

10. Which is true for a cyclic process?

(a) $\Delta E = 0$ (b) $\Delta E = q - W$
(c) $q = W$ (d) All of these

11. In the cannizzaro reaction given below

$$2C_6H_5CHO \xrightarrow{OH^-} C_6H_5CH_2OH + C_6H_5COO^-$$

The slowest step is

(a) The attack of OH^- at the carbonyl group
(b) The transfer of hydride ion to the carbonyl group
(c) The abstraction of proton from the carboxylic acid
(d) The deprotonation of benzyl alcohol

12. The rate of a reaction doubles when the initial concentration of the reactant is made four fold. If the initial concentration is made 400 fold, then the rate will become

(a) 400 times (b) 200 times
(c) 40 times (d) 20 times

13. Bisphenol and epichlorohydrin condensed in presence of NaOH forming

(a) Resins (b) Rubber
(c) Foam (d) Polyester

14. Which compound present in diesel?

(a) Cetane
(b) $TiCl_4$
(c) Cyclo pentandienyl manganese carbonyl
(d) Iso octane

15. The correct order of electron gain enthalpy with negative sign is

(a) S < O < Cl < F (b) O < S < F < Cl
(c) Cl < F < S < O (d) F < Cl < O < S

16. Which of the following is an organometallic compound?

(a) Lithium acetate
(b) Methyl lithium
(c) Lithium dimethyl amide
(d) Lithium methoxide

17. The total number of optical isomers possible in aldohexose is

(a) 2 (b) 16 (c) 8 (d) 4

18. Acetylene and HCHO react in presence of copper acetylide catalyst to form
(a) But-2-yne-1, 2-diol (b) But-1-yne-1, 4-diol
(c) But-2-ene-1, 4-diol (d) 2-butyne-1, 4-diol

19. If in the reaction $N_2O_4 \rightleftharpoons 2NO_2$; α is the degree of dissociation of N_2O_4, then total number of moles at equilibrium is
(a) $(1-\alpha)$ (b) $(1+\alpha)$
(c) $(1-\alpha)^2$ (d) $(1-\alpha)^2$

20. To convert 94 g of ice at 0°C into 94g of vapour at 100° C, the quantity of coal (90% carbon) required is : (ΔH_c of carbon = – 94 k cal mol^{-1}); latent heat of fusion = + 80 cal per gm; latent heat of vaporisation = + 540 cal per gm; specific heat of water = 1 cal per gm per °C)
(a) 94g (b) 80g
(c) 9.4g (d) 9.6g

21. Correct order of paramagnetism is
(a) Mn > Cr > Zn (b) Fe > Zn > Cr
(c) Cr > Fe > Zn (d) Zn > Mn > Fe

22. $CHCl_3 \xrightarrow[hv]{[O]} A \xrightarrow{NH_3} B \xrightarrow[PoCl_3]{\text{Malonic acid}} C$
The end product C in the following reaction is used as
(a) explosive (b) hypnotic
(c) tear gas (d) analgesic

23. The rate constant of a reaction is given by $\frac{N_t}{N_o} = e^{-kt}$. This represents for reaction of
(a) zero order (b) second order
(c) first order (d) None of these

24. Which statement is false for white phosphorus (P_4)?
(a) It has six P—P single bonds
(b) It has four P—P single bonds
(c) It has four lone pairs of electrons
(d) It has PPP angle 60°

25. Compound A on ozonolysis gives acetone and glyoxal. The compound A is
(a) 2, 5-dimethyl hex-2, 4-diene
(b) 2, 5-dimethyl hex-1, 5-diene
(c) 2, 5-dimethyl hex-3, 4-diene
(d) 2, 5-dimethyl but-2, ene

26. Natural rubber is a polymer of monomer isoprene. During polymerisaton
(a) 1, 4 addition takes place
(b) 1, 2 addition takes place
(c) 1, 3 addition takes place
(d) both double bonds are converted into single bond

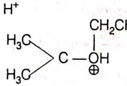
27. Select the correct statement.
(a) Orlon is an addition polymer
(b) Dacron is an addition polymer
(c) Orlon is a condensation polymer
(d) The monomers of orlon is styrene

28. Match List I (species) with List II (hybridization) and select the correct code given below

	List I		List II
A.	XeF_4	(i)	dsp^2
B.	H_2O	(ii)	sp^3
C.	PCl_5	(iii)	sp^3d^2
D.	$[Pt(NH_3)_4]^{2+}$	(iv)	sp^3d

Codes
A B C D A B C D
(a) (iii) (ii) (iv) (i) (b) (i) (iv) (ii) (iii)
(c) (iii) (iv) (ii) (i) (d) (i) (ii) (iv) (iii)

29. In acidic medium, the equivalent weight of $K_2Cr_2O_7$ (Mol. wt. = M) is
(a) M (b) $M/2$
(c) $M/3$ (d) $M/6$

30. Consider the following reaction
$Me_2C{=}CH_2 + H^+ \xrightleftharpoons{H_2SO_4} (X) \xrightleftharpoons{EtOH} (Y) \xrightarrow[(Z)]{} Me_3COEt.$
In this reaction which is not correct?
(a) (X) is Me_3C^+
(b) (X) is $Me_3CH{-}\overset{+}{C}H_2$
(c) (Z) is H^+
(d) (Y) is 

31. A new carbon-carbon bond formation is possible in :
I. Cannizzaro reaction
II. Friedel-Craft reaction
III. Clemmensen reduction
IV. Reimer-Tiemann reaction
(a) I, II and III (b) II, III and IV
(c) I and III (d) II and IV

32. 10 g of glucose (π_1), 10 g of urea (π_2) and 10 g of sucrose (π_3) are dissolved in 250mL of water at 300K. Correct order of osmotic pressure of solutions is
(a) $\pi_1 > \pi_2 > \pi_3$ (b) $\pi_3 > \pi_1 > \pi_2$
(c) $\pi_2 > \pi_1 > \pi_3$ (d) $\pi_2 > \pi_3 > \pi_1$

33. Which of the following compounds formed during Perkin's reaction?
(a) Resorcinol (b) Cinnamic acid
(c) Benzaldehyde (d) Benzoin

34. $^{235}_{92}U$ belongs to IIIB group of the Periodic When it loses one α-particle, the new element will belong to the group
(a) I B (b) V B (c) I A (d) III B

35. The ratio of de-Broglie wavelengths for electron accelerated through 200 V and 50 V is
(a) 1 : 2 (b) 2 : 1 (c) 3 : 10 (d) 10 : 3

36. Which set of elements have nearly the same atomic radii ?
(a) F, Cl, Br, I (b) Na, K, Rb, Cs
(c) Li, Be, B, C (d) Fe, Co, Ni, Cu

37. $CH_3CH_2C{\equiv}CH \underset{B}{\overset{A}{\rightleftharpoons}} CH_3C{\equiv}C{—}CH_3$, A and B are
(a) alc. KOH and SeO_2
(b) $NaNH_2$ and Lindlar catalyst
(c) alc. KOH and $NaNH_2$
(d) Lindlar catalyst and $NaNH_2$

38. Malachite decomposed to give $A + CO_2 + H_2O$ and compound A on reduction with carbon gives $CO + B$. Here, A and B are
(a) CuO, Cu (b) Cu_2O, CuO
(c) Cu_2O, Cu (d) $CuCO_3$, Cu

39. Match List I with List II and choose correct answer from the codes given below

List I	List II
A. $NaNO_3$	(i) Baking soda
B. $Na(NH_4)HPO_4$	(ii) Chile salt petre
C. $NaHCO_3$	(iii) Microcosmic salt
D. $Na_2CO_3 \cdot 10H_2O$	(iv) Washing soda

A B C D A B C D
(a) (i) (ii) (iii) (iv) (b) (ii) (iii) (i) (iv)
(c) (iii) (i) (ii) (iv) (d) (iv) (i) (ii) (iii)

40. When MnO_2 is heated with PbO_2 and conc. HNO_3, pink colour is obtained due to formation of
(a) $KMnO_4$ (b) $HMnO_4$ (c) $Pb(MnO_4)_2$ (d) $PbMnO_4$

41. The nuclear reaction, $^{27}_{13}Al + ^{4}_{2}He \rightarrow ^{30}_{14}Si + ^{1}_{1}H$ is
(a) nuclear fusion (b) nuclear fission
(c) nuclear transmutation (d) artificial radioactivity

42. Which is mismatched for NaCl crystal ?
(a) $\frac{r^+}{r^-} = 0.414$ to 0.732
(b) Coordination number = 6:6
(c) Edge of unit cell = $(r^+ + r^-)$
(d) Crystal structure = fcc

43. Which of the following ions has the highest magnetic moment ?
(a) Zn^{2+} (b) Ti^{3+} (c) Sc^{3+} (d) Mn^{2+}

44. "ELEKTRON" is an alloy of
(a) Cu, Zn and Mg (b) Fe and Mg
(c) Ni and Zn (d) Al and Zn

45. The structure of ionic compound A^+B^- is identical to NaCl. If the edge length is 400pm and cation radius is 75pm, the radius of anion will be
(a) 100 pm (b) 125 pm (c) 250 pm (d) 325 pm

46. Which of the following statements is true?
(a) $SnCl_4$ is more stable than $SnCl_2$
(b) $PbCl_2$ is more stable than $PbCl_4$
(c) $GeCl_4$ is more stable than $GeCl_2$
(d) $TlCl_3$ is more stable than TlCl

47. Correct order for solubility of alkaline earth metals in water is
(a) $MgF_2 > CaF_2 > SrF_2 > BaF_2$
(b) $MgF_2 < CaF_2 < SrF_2 < BaF_2$
(c) $MgF_2 > CaF_2 < SrF_2 < BaF_2$
(d) $BaF_2 > MgF_2 > SrF_2 > CaF_2$

48. An organic compound A contains 20% C, 46.66% N and 6.66% H. It gives NH_3 gas on heating with NaOH. A can be
(a) CH_3CONH_2 (b) $C_6H_5CONH_2$
(c) NH_2CONH_2 (d) $CH_3NHCONH_2$

49. A metal reacts with dil acid and liberates hydrogen. If the reduction potential of hydrogen be considered zero, the reduction potential of that metal will be
(a) equal to its oxidation potential
(b) positive
(c) zero
(d) negative

50. In the equilibrium mixture, $KI + I_2 \rightleftharpoons KI_3$; the concentration of KI and I_2 is made two fold and three fold respectively . The concentration of KI_3 becomes
(a) two fold (b) three fold
(c) five fold (d) six fold

Mathematics

1. The angle between planes $2x-y+z=6$ and $x+y+2z=3$ is

(a) 30° (b) 60° (c) $\cos^{-1}\sqrt{\frac{3}{2}}$ (d) $\sin^{-1}\sqrt{\frac{3}{2}}$

2. Equation of tangent to the circle $x^2+y^2-2x-2y+1=0$ perpendicular to $y=x$ is given by

(a) $x+y\pm 1=0$ (b) $x+y=2\pm\sqrt{3}$
(c) $x-y\pm 3=0$ (d) $x-y\pm 1=0$

3. If A and B are two such events that $P(A\cup B)=P(A\cap B)$, then which of the following is true?

(a) $P(A)+P(B)=0$
(b) $P(A)+P(B)=P(A)\,P(B/A)$
(c) $P(A)+P(B)=2P(A)\,P(B/A)$
(d) None of the above

4. The y-coordinate of a point P on the line joining $A(7,2,1)$ and $B(10,5,7)$ is 4. Then, x and z-coordinates of the point are

(a) $x=9,\ z=5$ (b) $x=3,\ z=7$
(c) $x=2,\ z=3$ (d) None of these

5. If point D divides the base BC of a ΔABC in the ratio $n:m$, then the value of $mBD^2+nCD^2+(m+n)AD^2$ is

(a) mAC^2+nAB^2 (b) $(m+n)(AC^2+AB^2)$
(c) nAC^2+mAB^2 (d) None of these

6. A man is standing on the horizontal plane. The angle of elevation of top of the pole is α. If he walks a distance double the height of the pole, then the elevation of the pole is 2α. The value of α is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

7. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]} & \text{for } [x]\neq 0\\ 0 & \text{for } [x]=0\end{cases}$

where, $[x]$ denotes the greatest integer less than or equal to x, then $\lim\limits_{x\to 0} f(x)$ is equal to

(a) 1 (b) -1
(c) 0 (d) Does not exist

8. The value of the angle between two straight lines $y=(2-\sqrt{3})x+5$ and $y=(2+\sqrt{3})x-7$ is

(a) 30° (b) 60°
(c) 45° (d) 90°

9. Which of the following is a universal gate?

(a) NAND (b) OR (c) AND (d) NOT

10. According to Simpson's rule, the value of $\int_1^7 \frac{dx}{x}$ is

(a) 1.358 (b) 1.958
(c) 1.625 (d) 1.458

11. Equation of a plane passing through $(-1,1,1)$ and $(1,-1,1)$ and perpendicular to $x+2y+2z=5$ is

(a) $2x+3y-3z+3=0$ (b) $x+y+3z-5=0$
(c) $2x+2y-3z+3=0$ (d) $x+y+z-3=0$

12. The value of $\sin 50^\circ\cos 10^\circ+\cos 50^\circ\sin 10^\circ$ is

(a) $\frac{1}{2}$ (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

13. If $f'(x)>0\ \forall\ x\in R, f'(3)=0$ and $g(x)=f(\tan^2 x-2\tan x+4), 0<x<\frac{\pi}{2}$, then $g(x)$ is increasing in

(a) $\left(0,\frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ (c) $\left(0,\frac{\pi}{3}\right)$ (d) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

14. Probability of getting a total of 7 or 9 in a single throw of two dice is

(a) $\frac{5}{18}$ (b) $\frac{1}{6}$
(c) $\frac{1}{9}$ (d) None of these

15. What is compiler?

(a) Application software (b) System software
(c) Utility software (d) All of these

16. If $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $fog(x)$ is equal to

(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) None of these

17. Let $f(x)$ be differentiable on the interval $(0,\infty)$ such that $f(1)=1$ and $\lim\limits_{t\to x}\frac{t^2 f(x)-x^2 f(t)}{t-x}=1$ for each $x>0$. Then, $f(x)$ is equal to

(a) $\frac{1}{3x}+\frac{2}{3}x^2$ (b) $-\frac{x}{3}+\frac{4x^2}{3}$
(c) $-\frac{1}{x}$ (d) $-\frac{1}{x}+\frac{2}{x^2}$

18. The position vectors of three non-collinear points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, respectively. The perpendicular distance of point C from the straight line AB is

(a) $\frac{|\mathbf{b}\times\mathbf{c}|}{|\mathbf{b}-\mathbf{c}|}$ (b) $\frac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{b}-\mathbf{a}|}$
(c) $\frac{|\mathbf{c}\times\mathbf{a}|}{|\mathbf{c}-\mathbf{a}|}$ (d) $\frac{|\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}|}{|\mathbf{b}-\mathbf{a}|}$

19. If $\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

$=A\sin^{-1}x+Bx\sqrt{1-x^2}+C,$

then $A+B$ is equal to

(a) 10 (b) $\frac{1}{2}$
(c) 1 (d) $-\frac{1}{2}$

20. The coefficient of x^4 in $(1+x+x^3+x^4)^{10}$ is

(a) 210 (b) 100
(c) 310 (d) 110

21. If $A=\begin{bmatrix}4 & 11\\ 2 & 6\end{bmatrix}$, then A^{-1} is equal to

(a) $\begin{bmatrix}-1 & \frac{-11}{2}\\ 3 & 2\end{bmatrix}$ (b) $\begin{bmatrix}3 & \frac{-11}{2}\\ -1 & 2\end{bmatrix}$

(c) $\begin{bmatrix}3 & 2\\ -1 & \frac{-11}{2}\end{bmatrix}$ (d) None of these

22. The locus of centre of circles which cuts orthogonally the circle $x^2+y^2-4x+8=0$ and touches $x+1=0$, is

(a) $y^2+6x+7=0$
(b) $x^2+y^2+2x+3=0$
(c) $x^2+3y+4=0$
(d) None of the above

23. Let $f(x)=\begin{vmatrix}\sin 3x & 1 & 2\left(\cos\frac{3x}{2}+\sin\frac{3x}{2}\right)^2\\ \cos 3x & -1 & 2\left(\cos^2\frac{3x}{2}-\sin^2\frac{3x}{2}\right)\\ \tan 3x & 4 & 1+2\tan 3x\end{vmatrix}$

Then, the value of $f'(x)$ at $x=(2n+1)\pi, n\in I$ (the set of integers) is equal to

(a) $(-1)^n$
(b) $(-1)^{n+1}$
(c) 3
(d) 9

24. The condition for the line $lx+my+n=0$ to be a normal to $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

(a) $\frac{l^2}{9}+\frac{m^2}{25}=\frac{n^2}{256}$ (b) $\frac{9}{m^2}+\frac{25}{l^2}=\frac{256}{n^2}$
(c) $\frac{l^2}{9}-\frac{m^2}{25}=\frac{n^2}{256}$ (d) None of these

25. The least value of a, for which the function $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ has atleast one solution in the interval $\left(0,\frac{\pi}{2}\right)$, is

(a) 9 (b) 4 (c) 5 (d) 1

26. If one line of regression coefficient is less than unity, then the other will be

(a) less than unity (b) equal to unity
(c) greater than unity (d) All of these

27. Three concurrent edges of a parallelopiped are given by

$\mathbf{a}=2\hat{i}-3\hat{j}+\hat{k},$
$\mathbf{b}=\hat{i}-\hat{j}+2\hat{k}$
and $\mathbf{c}=2\hat{i}+\hat{j}-\hat{k}.$

The volume of the parallelopiped is

(a) 14 cu units (b) 20 cu units
(c) 25 cu units (d) 60 cu units

28. Roots of equation $x^3-6x+1=0$ lie in the interval

(a) (2, 3) (b) (3, 4)
(c) (3, 5) (d) (4, 6)

29. RAM is a

(a) volatile memory
(b) non-volatile memory
(c) cash memory
(d) dynamic memory

30. If $\lim\limits_{n\to\infty}\sum\frac{\log(n+r)-\log n}{n}=2\left(\log 2-\frac{1}{2}\right)$, then $\lim\limits_{n\to\infty}\frac{1}{n^\lambda}[(n+1)^\lambda(n+2)^\lambda\ldots(n+n)^\lambda]^{1/n}$ is equal to

(a) $\frac{4\lambda}{e}$ (b) $\left(\frac{4}{e}\right)^\lambda$ (c) $\left(\frac{4}{e}\right)^{\frac{1}{\lambda}}$ (d) $\left(\frac{e}{4}\right)^\lambda$

31. If $\lim\limits_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=\frac{-1}{12}$, then

(a) $a=2, b\in R, c=0$ (b) $a=-2, b\in R, c=0$
(c) $a=1, b\in R, c=0$ (d) $a=-1, b\in R, c=0$

32. According to Newton-Raphson method, the value of $\sqrt{12}$ upto three places of decimal will be

(a) 3.463 (b) 3.462
(c) 3.467 (d) None of these

33. If $\frac{(3-i)^2}{2+i} = A + iB$, where A and B are real numbers, then A and B are equal to

(a) $A = -4, B = 2$
(b) $A = 2, B = -4$
(c) $A = 2, B = 4$
(d) None of these

34. The radical centre of the system of circles,

$$x^2 + y^2 + 4x + 7 = 0,$$
$$2(x^2 + y^2) + 3x + 5y + 9 = 0$$
and $$x^2 + y^2 + y = 0 \text{ is}$$

(a) $(-2, -1)$ (b) $(1, -2)$
(c) $(-1, -2)$ (d) None of these

35. The sum of n terms of the series $1 + 5 + 12 + 22 + 35 + \dots$ is

(a) $\frac{n^2(n+1)}{8}$ (b) $\frac{n^2(n+1)}{6}$
(c) $\frac{n^2(n+1)}{2}$ (d) None of these

36. The curve, for which the area of the triangle formed by X-axis, the tangent line at any point P and line OP is equal to a^2, is given by

(a) $y = x - Cx^2$ (b) $x = Cy \pm \frac{a^2}{y}$
(c) $y = Cx \pm \frac{a^2}{x}$ (d) None of these

37. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is defined by

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1 \pm \sqrt{1 + 4\log_2 x})$
(c) $\frac{1}{2}(1 - \sqrt{1 - 4\log_2 x})$ (d) None of these

38. Solution of the equation $\cos^2 x \frac{dy}{dx} - (\tan 2x) y = \cos^4 x$, $|x| < \frac{\pi}{4}$, where $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$, is given by

(a) $y \frac{\tan 2x}{1 - \tan^2 x} = 0$ (b) $y(1 - \tan^2 x) = C$
(c) $y = \sin 2x + C$ (d) $y = \frac{1}{2} \cdot \frac{\sin 2x}{1 - \tan^2 x}$

39. If lines of regression are $3x + 12y = 19$ and $3y + 9x = 46$, then value of r_{xy} will be

(a) 0.289
(b) −0.289
(c) 0.209
(d) None of these

40. If $1, \omega$ and ω^2 are the cube roots of unity, then the value of $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$ is equal to

(a) 4 (b) 0
(c) 2 (d) 3

41. The equation of the curve through the point $(1, 0)$, whose slope is $\frac{y-1}{x^2 + x}$, is

(a) $2x(y-1) + x + 1 = 0$
(b) $(x+1)(y-1) + 2x = 0$
(c) $x(y-1)(x+1) + 2 = 0$
(d) $x(y+1) + y(x+1) = 0$

42. The number of points, where $f(x) = [\sin x + \cos x]$ (where $[\cdot]$ denotes the greatest integer function) and $x \in (0, 2\pi)$ is not continuous, is

(a) 3 (b) 4 (c) 5 (d) 6

43. The value of $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$ is equal to

(a) $\frac{x}{3}$ (b) $\frac{x}{4}$
(c) 1 (d) $\frac{x}{2}$

44. If $A(-1, 3, 2), B(2, 3, 5)$ and $C(3, 5, -2)$ are vertices of a ΔABC, then angles of ΔABC are

(a) $\angle A = 90°, \angle B = 30°, \angle C = 60°$
(b) $\angle A = \angle B = \angle C = 60°$
(c) $\angle A = \angle B = 45°, \angle C = 90°$
(d) None of the above

45. $\int_{1/e}^{1} |\log x| \, dx$ is equal to

(a) $\frac{1}{e}$ (b) e
(c) $2\left(1 - \frac{1}{e}\right)$ (d) $2\left(1 + \frac{1}{e}\right)$

46. $\lim_{x \to 0} \frac{\int_0^{x^2} \sin \sqrt{t} \, dt}{x^3}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) 0 (d) ∞

47. If **a**, **b** and **c** are three non-coplanar vectors, then $[\mathbf{a} \times \mathbf{b} \; \mathbf{b} \times \mathbf{c} \; \mathbf{c} \times \mathbf{a}]$ is equal to

(a) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^3$ (b) $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^2$
(c) 0 (d) None of these

48. If geometric mean and harmonic mean of two numbers a and b are 16 and 64/5 respectively, then the value of $a : b$ is

(a) 4 : 1 (b) 3 : 2
(c) 2 : 3 (d) 1 : 4

49. The value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(a) 0 (b) $a + b + c$
(c) abc (d) 1

50. If the sum of four numbers in GP is 60 and the arithmetic mean of the first and last numbers is 18, then the numbers are

(a) 3, 9, 27, 81 (b) 4, 8, 16, 32
(c) 2, 6, 18, 54 (d) None of these

Answers

Physics

1. (a)	2. (c)	3. (c)	4. (b)	5. (a)	6. (b)	7. (a)	8. (d)	9. (c)	10. (c)
11. (c)	12. (b)	13. (c)	14. (d)	15. (d)	16. (b)	17. (b)	18. (b)	19. (c)	20. (d)
21. (b)	22. (d)	23. (d)	24. (c)	25. (d)	26. (a)	27. (b)	28. (b)	29. (b)	30. (b)
31. (a)	32. (a)	33. (b)	34. (b)	35. (b)	36. (c)	37. (a)	38. (b)	39. (c)	40. (a)
41. (d)	42. (a)	43. (b)	44. (a)	45. (a)	46. (a)	47. (c)	48. (a)	49. (c)	50. (a)

Chemistry

1. (c)	2. (a)	3. (a)	4. (a)	5. (c)	6. (c)	7. (a)	8. (c)	9. (c)	10. (a)
11. (b)	12. (d)	13. (a)	14. (a)	15. (b)	16. (b)	17. (b)	18. (d)	19. (b)	20. (d)
21. (c)	22. (b)	23. (c)	24. (b)	25. (a)	26. (a)	27. (a)	28. (a)	29. (d)	30. (b)
31. (d)	32. (c)	33. (b)	34. (d)	35. (a)	36. (d)	37. (c)	38. (c)	39. (b)	40. (b)
41. (d)	42. (c)	43. (d)	44. (d)	45. (b)	46. (b)	47. (a)	48. (c)	49. (d)	50. (a)

Mathematics

1. (b)	2. (*)	3. (c)	4. (a)	5. (c)	6. (a)	7. (d)	8. (b)	9. (a)	10. (b)
11. (c)	12. (c)	13. (d)	14. (a)	15. (b)	16. (b)	17. (a)	18. (d)	19. (c)	20. (c)
21. (b)	22. (a)	23. (c)	24. (b)	25. (a)	26. (c)	27. (a)	28. (a)	29. (a)	30. (b)
31. (b)	32. (d)	33. (b)	34. (a)	35. (c)	36. (b)	37. (d)	38. (d)	39. (b)	40. (a)
41. (*)	42. (c)	43. (d)	44. (d)	45. (*)	46. (a)	47. (b)	48. (a)	49. (a)	50. (b)

(*) None of the option is correct.

Hints & Solutions

Physics

1. Surface tension, $S = \dfrac{\text{Work done}}{\text{Increase in area}}$

$$= \frac{2\times 10^{-4}}{2\,(10\times 6-8\times 3.75)\times 10^{-4}}$$

$$= 0.033$$

$$= 3.3\times 10^{-2}\ \text{Nm}^{-1}$$

2. Increase in potential energy

= Final potential energy − Initial potential energy

$$= -\frac{GMm}{\left(R+\frac{R}{5}\right)} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{5}{6}\frac{GMm}{R} + \frac{GMm}{R}$$

$$= \frac{1}{6}\frac{GMm}{R} = \frac{1}{6}\frac{GMmR}{R^2}$$

$$= \frac{1}{6}mgR \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{5}{6}mgh \qquad \left[\because h = \frac{R}{5}\right]$$

3. Potential energy, $U = \dfrac{1}{2}\dfrac{F^2}{K}$

$\Rightarrow \quad U \propto \dfrac{1}{K} \qquad [\because F \text{ is same}]$

$\therefore \quad \dfrac{U_1}{U_2} = \dfrac{K_2}{K_1} = \dfrac{1000}{2000} = \dfrac{1}{2} = 1:2$

4. To break the covalent bond in a semiconductor, energy equal to forbidden energy gap is required.

5. Random and excess exposure to X-rays may induce diseases and has a damaging effect on the living cells of a body.

6. In a potentiometer experiment, if deflection in galvanometer is measured zero, then current in galvanometer circuit will also become zero.

7. Given, $A = 60°$ and $\delta_m = 30°$

So, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}} = \dfrac{\sin\left(\dfrac{60°+30°}{2}\right)}{\sin\dfrac{60°}{2}}$

$$= \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}}\times 2 = \frac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

We have, $\mu = \dfrac{1}{\sin C}$

or $\sqrt{2} = \dfrac{1}{\sin C}$ or $C = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

8. The electric field (E) inside a uniformly charged conducting spherical shell is zero at all points.

9. Faraday constant is a universal constant.

10. Interference effect can be observed in both transverse wave and longitudinal wave.

11. If a comb is run through wet hair on rainy day, then it will not attract small bits of paper.

12. Given, $x = 9t^2$

$\therefore \quad v = \dfrac{dx}{dt} = \dfrac{d}{dt}(9t^2) = \dfrac{9d}{dt}(t^2) = 18t$

also, $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = \dfrac{d}{dt}(18t) = 18$

So, the graph between acceleration and time is a line parallel to time axis as acceleration is constant.

13. From first law of thermodynamics,

$$dQ = dU + dW$$

For a isochoric process, $dW = 0$

$\therefore \quad dQ = dU$

$\therefore$ Amount of heat required, $dQ = nC_V\,(T_2 - T_1)$

$$= n\frac{C_V}{C_p}C_p\,(T_2-T_1)$$

$$= 1\times\gamma\times 2.5\times 8.3\times(423-273)$$

$$= \frac{3}{5}\times 2.5\times 8.3\times 150$$

$$= 1867.5\ \text{J}$$

14. The minimum negative potential V_0 is called stopping potential.

Here, $K_{max} = 10$ eV

So, the stopping potential, $eV_0 = K_{max}$

$\Rightarrow \quad eV_0 = 10$ eV

or $\quad V_0 = -10$ V

15. Tangent galvanometer is not useful to measure current as it is not directly readable, at different places different readings are obtained and if there is any magnetic material near it, it will effects its readings.

16. Thermal radiations exists in infrared part of the electromagnetic spectrum.

17. When two similar cells are connected in series.

The effective emf, $E_1 = E + E = 2E$

When two similar cells are connected in parallel.

Then, the effective emf $E_2 = E$

According to potentiometer wire,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \Rightarrow \frac{2E}{E} = \frac{l_1}{l_2}$$

Hence, $l_1 : l_2 = 2:1$

18. Given, $p = \dfrac{a-t^2}{bx}$

$\Rightarrow \quad p = \dfrac{a}{bx} - \dfrac{t^2}{bx}$

By the principle of homogeneity

$$p = \frac{a}{bx} \Rightarrow \frac{a}{b} = px$$

or $\quad \left[\dfrac{a}{b}\right] = [p]\,[x]$

$$= [ML^{-1}T^{-2}]\,[L] = [MT^{-2}]$$

19. For a thin uniform rod, moment of inertia about an axis through its centre perpendicular to length of rod, $I = \dfrac{1}{12}ML^2$

Here, $\quad M = (m_w + m_b)$

$\therefore \quad I = \dfrac{1}{12}(m_w + m_b)L^2$

20. The time period of a particle executing SHM is given as

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$= 2\pi\sqrt{\frac{3\times 10^{-2}}{12\times 10^{-2}}} = 2\pi\times\frac{1}{2}$$

$$= \pi = 3.14\ \text{s}$$

21. The value of gravitational acceleration at equator due to rotation of the earth is

$$g_{eq} = g - R\omega^2$$

If the earth stops rotating, then $g_{eq} = g$

So, change in g, $\Delta g = R\omega^2 = 3.41\times 10^{-2}\ \text{ms}^{-2}$

Hence % increase in $g = \dfrac{\Delta g}{g}\times 100$

$$= \frac{3.41\times 10^{-2}}{9.8}\times 100$$

$$= 0.3\%$$

22. If the moving charged particle enters in a magnetic field making some angle with its initial direction of motion, then it will describes a helical path.

23. A transformer is essentially an AC device, it does not work on DC. So, voltage developed across secondary is zero.

24. In the state of minimum deviation the refracted ray becomes parallel to the base of the prism. So, angle of incidence is less than angle of refraction.

25. Here path difference $= \dfrac{\lambda}{2}$

$\therefore$ Phase difference $= \dfrac{2\pi}{\lambda}\times$ path difference

i.e., $\phi = \dfrac{2\pi}{\lambda}\times\dfrac{\lambda}{2} = \pi$

So, maximum intensity,

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\pi$$

or $\quad I_{max} = I + I + 2\sqrt{I\,I}\,(-1) \qquad [\because I_1 = I_2]$

$$I_{max} = 0$$

26. Resolving power of eye $= \left(\dfrac{1}{60}\right)^{\circ} = \dfrac{1}{60}\times\dfrac{\pi}{180}$

Let the minimum distance between the poles be d,

then, $\quad \dfrac{d}{11000} = \dfrac{1}{60}\times\dfrac{\pi}{180}$

or $\quad d = 11000\times\dfrac{1}{60}\times\dfrac{\pi}{180}$

$$= 3\ \text{m}$$

27. Stefan's constant, $\sigma = \dfrac{E}{T^4}$ (where, E is energy/s area)

$$= \frac{[ML^2T^{-2}]}{[T]\,[L^2]\,[\theta^4]}$$

$$= [MT^{-3}\theta^{-4}]$$

28. Here, $e = 8\,V$

$$dI = (4-2)\,A = 2A$$
$$dt = 0.05\,s$$

As, $$e = L\frac{dI}{dt}$$

$\therefore$ $$L = \frac{e\,(dt)}{dI} = \frac{8\times(0.05)}{5} = 0.2\,H$$

29. Here, the force is acting along y-direction.

$\therefore$ The work done in displacing a body from $y = -a$ to $y = +a$

$$W = \int_{-a}^{+a} \mathbf{F}\cdot d\mathbf{y} = \int_{-a}^{+a} F\,dy$$

$\left[\because \text{force and displacement are in same direction}\right]$

$$= \int_{-a}^{+a} (Ay^2 + By + C)\,dy$$
$$= \int_{-a}^{+a} (Ay^2)\,dy + \int_{-a}^{+a} (By)\,dy + \int_{-a}^{+a} C\,dy$$
$$= \frac{2Aa^3}{3} + 2Ca$$

30. Time period of oscillation of mass m suspended from a spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

If the spring is cut into two halves, then the new time period.

$$T' = 2\pi\sqrt{\frac{m}{2k}} = 2\frac{\pi}{\sqrt{2}}\sqrt{\frac{m}{k}} = \frac{T}{\sqrt{2}}$$

31. Given, $I = 2.0\,A$

$$R = 25\,\Omega$$
$$t = 1\,\text{min} = 60\,s$$

$\therefore$ Heat produced, $H = I^2 Rt$

$$= (2)^2 \times 25 \times 60$$
$$= 4 \times 25 \times 60\,J$$
$$= 6\times 10^3\,J$$

32. Specific resistance depends on the temperature. For metals $\rho_t = \rho_0(1+\alpha\Delta t)$ *i.e.*, specific resistance of a conductor increases with rise in temperature.

33. The radius of nth orbit of hydrogen like an ion,

$$r_n = \frac{n^2 h^2}{4\pi^2\,kmze^2}$$

where, $\frac{h^2}{4\pi^2 kme^2} = a_0$ (Bohr's radius)

then, $$r_n = \frac{a_0 n^2}{z}$$

34. Frequency for a mass m executing SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{...(i)}$$

When the mass is increased by 4 times, then the new frequency

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{1}{2}\cdot\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{f}{2}$$

35. The capacitance of a capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

$\Rightarrow$ $$\varepsilon_0 = \frac{Cd}{A} = \frac{\text{farad}\times\text{meter}}{(\text{meter})^2} = \text{farad/meter}$$

36. As, the final image in formed at infinity by the eye lens, so the object distance for eye lens is focal length of eye lens.

So, $u_e = 6\,\text{mm} = 0.6\,\text{cm}$

The distance between the objective and eye lens is 20.6 cm.

$\therefore$ $L = v_0 + u_e \Rightarrow v_0 = L - u_e$

$$= 20.6 - 0.6 = 20\,\text{cm}$$

Let u be the object distance form objective, then from lens formula for objective.

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0} \Rightarrow \frac{1}{u_0} = -\frac{1}{f_0} + \frac{1}{v_0} = \frac{1}{20} - \frac{1}{0.6} = -\frac{19.4}{12}$$

or $$u_0 = -\frac{12}{19.4} = -0.62\,\text{cm}.$$

So, the magnification of objective,

$$m = \frac{v_0}{u_0} = \frac{20}{-0.62} = -32.3$$

37. For isothermal process, $dU = 0$,

i.e., internal energy remains unchanged.

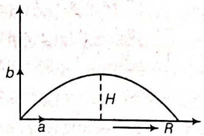
For a cyclic process, $dU = 0$,

i.e., internal energy remains unchanged.

38. Given, $\mathbf{u} = (a\hat{i} + b\hat{j})$ m/s

So, $u\cos\theta = a$

and $u\sin\theta = b$



According to the question,

$$R = 2H$$

$\Rightarrow$ $$\frac{u^2\sin 2\theta}{g} = \frac{2\times u^2\sin^2\theta}{2g}$$

$\Rightarrow$ $$\frac{u^2\cdot 2\sin\theta\cdot\cos\theta}{g} = \frac{2u^2\sin^2\theta}{2g}$$

$\Rightarrow$ $$2 = \frac{b}{a}$$

$\Rightarrow$ $$b = 2a$$

39. An electric charge in uniform motion produces both electric field and magnetic field.

40. During charging and discharging of a capacitor through resistor the time constant, $\tau = RC$.

41. Time period,

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

$\therefore$ $$\frac{H_1}{H_2} = \frac{T_2^2}{T_1^2}$$

Here, $$T_1 = \frac{5\times 60}{25} = 12\,s$$

and $$T_2 = 9\,s$$

$\therefore$ $$\frac{H_1}{H_2} = \frac{(9)^2}{(12)^2} = \frac{9}{16}$$

42. For photoelectric effect to take place, the incident wavelength (λ) should be equal to or smaller than the threshold wavelength (λ_0) *i.e.*,

$$\lambda \le \lambda_0$$

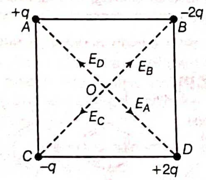
43. $${}_7X^{15} + {}_2He^4 \longrightarrow {}_9Y^{19}$$
$${}_9Y^{19} \longrightarrow {}_8Z^{18} + {}_1H^1$$

According to the conservation of mass number

$$19 = 18 + 1$$

So, the mass number of changed atom = 18

44. $$AD = BC = \sqrt{(5)^2 + (5)^2} = \sqrt{25+25} = \sqrt{2}\times 5\,\text{cm}$$



$\Rightarrow$ $AO = BO = CO = OD = \frac{5\sqrt{2}}{2}$ cm

The electric field,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

So, $$E_A = \frac{9\times10^9\times10\times10^{-9}\times4}{25\times2} = 7.2\ \text{NC}^{-1}\ \text{along } OD$$

$$E_B = \frac{9\times10^9\times2\times10\times10^{-9}\times4}{25\times2} = 14.4\,\text{NC}^{-1}\ \text{along } OB$$

$$E_C = \frac{9\times10^9\times10\times10^{-9}\times4}{25\times2} = 7.2\ \text{along } OC$$

$$E_D = \frac{9\times10^9\times2\times10\times10^{-9}\times4}{25\times2} = 14.4\ \text{along } OA$$

Resultant of E_A and E_D, $E_1 = (14.4 - 7.2) = 7.2\,\text{NC}^{-1}$ along OA.

Resultant of E_B and E_C, $E_2 = (14.4 - 7.2) = 7.2\,\text{NC}^{-1}$ along OB.

Since, E_1 and E_2 are perpendicular to each other.

$\therefore$ $E = \sqrt{E_1^2 + E_2^2}$ is along 45° to OA upward.

45. For an adiabatic process, $TV^{\gamma-1}$ = constant.

46. The value of resistance connected in series to galvanometer $= (n-1)G$.

47. The correct relation is $v_{rms} > \bar{v} > v_{mp}$

As $\bar{v} = 1.6\sqrt{\frac{RT}{M}}$, $v_{rms} = 1.73\sqrt{\frac{RT}{M}}$

and $v_{mp} = 1.41\sqrt{\frac{RT}{M}}$

48. Due to reverse biasing, in a junction diode the potential barrier increases.

49. Biot-Savart's law in vector form

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}\hat{n}$$

50. Given, Vp^2 = constant

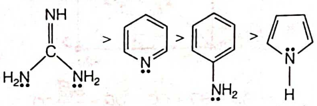
or $V\left[\frac{RT}{V}\right]^2$ = constant or $\frac{T^2}{V}$ = constant

or $\frac{T'^2}{T^2} = \frac{V'}{V}$

$\Rightarrow \frac{T'^2}{T^2} = \frac{2V}{V} \Rightarrow T' = \sqrt{2}T$

Chemistry

1.



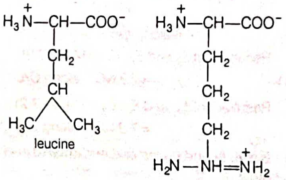
From the structures of the four compounds, It is clear that in pyrrole, the electron pair on nitrogen is involved in delocalisation of the ring. So, it is least basic.

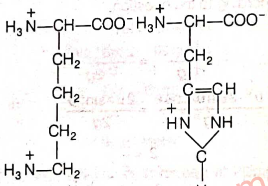
In aniline, the electron pair involved in conjugation with the π^- electrons of the ring while in pyridine it is relatively free. Guanidine is a strong base.

2. Lithium is the strongest reducing agent. However IE of lithium is highest among alkali metals. So, its reducing nature should be minimum.

The greatest reducing nature of Li in aqueous medium is due to the maximum hydration energy of Li^+ ion.

3.





Leucine is not a basic amino acid while others three *i.e.*, arginine, lysine and histidine are basic amino acids as they have more number of amino groups than carboxyl groups.

4. The extent of adsorption increases with the increase of surface area of adsorbent. Finely divided charcoal is a good adsorbent.

5. In qualitative analysis, NH_4Cl is added before NH_4OH to decrease $[OH^-]$ concentration. NH_4Cl suppresses the ionisation of NH_4OH due to common ion effect. Otherwise basic radicals of group V and group VI will be precipitated along with IIIrd group basic radicals.

6. $C_2H_5Cl \xrightarrow[CuI]{Li} (C_2H_5)_2CuLi \xrightarrow{C_2H_5Cl} \underset{n\text{-butane}}{CH_3CH_2CH_2CH_3}$

The reaction is known as Corey House synthesis for alkanes. (symmetrical and unsymmetrical both)

7. 11.2 L H_2 gas at NTP = 0.5 mole of H_2 gas = 1 g of H_2 g

$\frac{W_1}{W_2} = \frac{E_1}{E_2} = \frac{1}{W_2} = \frac{1}{108}$ or $W_2 = 108$g silver is deposited in half an hour.

Therefore, $108 \times 2 = 216$ g silver will be deposited in one hour.

8. A. $[Co(NH_3)_6]^{+3}$ inner orbital (d^2sp^3) complex

B. $[Fe(CN)_6]^{4-}$ inner orbital (d^2sp^3) complex

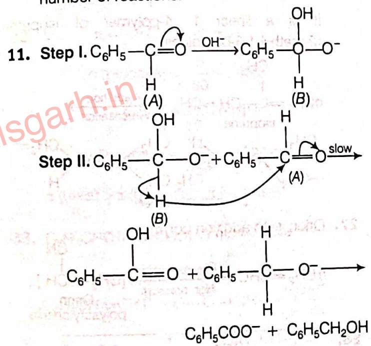
C. $[Ni(NH_3)_6]^{2+}$ outer orbital (sp^3d^2) complex

D. $[Mn(CN)_6]^{2-}$ inner orbital (d^2sp^3) complex

9. $K_3[Fe(CN)_6]$ Potassium hexacyanoferrate (III)

10. For a cyclic process, $\Delta E = 0$. In cyclic process, system returns to its original state after a number of reactions.

11.



H^- is obtained by fission of C—H bond of (*B*). It is transferred to (II). This is the slowest step.

12. $R = k[A]^x$ ...(i)

$2R = k[4A]^x$ ...(ii)

Divide Eq. (ii) by (i)

$2 = [4]^x$

$R = k[A]^{1/2}$

$x = \frac{1}{2}$

or $= 0.5$

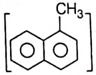
$R = k(400\ A)^{1/2}$

Rate $= 20k[A]^{1/2}$

Rate becomes 20 times.

13. Epoxy resins are obtained by copolymeristation of epichlorohydrin and bisphenol *A*. These resins have good adhesive strength.

14. Diesel is fraction of petroleum having $C_{15}-C_{18}$ composition and boiling point in the range of 573 – 673 K. Generally cetane $[CH_3-(CH_2)_{14}-CH_3]$ and α-methyl naphthalene



are its main constituents.

15. Order of electron gain enthalpy is as follows O < S < F < Cl. Because of the compact nature of oxygen and fluorine atoms, they have less negative electron gain enthalpy than S and Cl respectively .

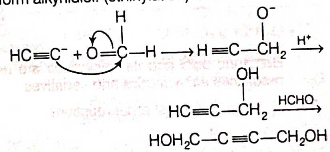
Due to the small size of oxygen and fluorine atoms, there are strong interelectronic repulsions in the relatively small 2 *p*-orbitals of oxygen and fluorine and thus incoming electron does not experience much attraction.

16. Organometalllic compounds must contain, at least one metal-carbon bond. Methyl lithium, CH_3Li is an organometallic compound.

17. Aldohexose has 4 chiral carbon atoms.

No. of optical isomers $= 2^n = 2^4 = 16$

18. Termimal alkynes (Containing C≡CH, *i.e.*, a methine hydrogen atom) can add on to carbonyl group in the presence of catalyst to form alkynidiol. (ethinylation)



19. $N_2O_4 \rightleftharpoons 2NO_2$

	N_2O_4	$2NO_2$
Initial	1	0
At. equil.	1	0

Total number of moles at equilibrium

$= (1-\alpha) + 2\alpha$

$= 1 + \alpha$

20. $q = m\,s\,\Delta t$

$0°C$ Ice (s) $\longrightarrow$ $0°C$ water (l)

$q_1 = 94 \times 80 = 7520$ cal

$0°C$ water (l) $\longrightarrow$ $100°C$ water (l)

$q_2 = 94 \times 1 \times 100 = 9400$ cal

$100°C$ (water) $100°C$ water vapours

$q_3 = 94 \times 540 = 50760$ cal

Total (q) = 67680cal

= 67.680 kcal

$\because$ 94 kcal = 12 gcoal

$\therefore$ $67.68\text{kcal} = \dfrac{67.68 \times 12}{94}$

= 8.64g cal

90% of x = 8.64

or x = 9.6 g cal

21. Paramagnetic character $\propto$ number of unpaired electrons

$Cr = 3d^5 4s^1$; $6e^-$

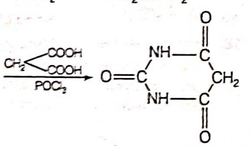
$Mn = 3d^5 4s^2$; $5e^-$

$Zn = 3d^{10} 4s^2$; No

$Fe = 3d^6 4s^2$; $4e^-$

Hence, the correct order of paramagnetism is $Cr > Mn > Fe > Zn$.

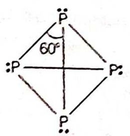
22. $CHCl_3 \xrightarrow[h\nu]{[O]} COCl_2 \xrightarrow{NH_3} NH_2CONH_2$



barbituric acid

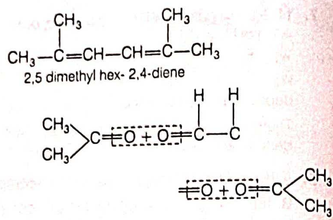
Barbituric acid and its derivatives are used in medicines as hypnotics and sedatives.

23. $N_t = N_0 e^{-\lambda t}$ for first order reactions.

24. 

White phosphorus (P_4)

It has six P—P single bonds.

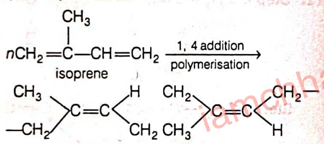
25. 

2,5 dimethyl hex- 2,4-diene

To decide the structure of alkane from ozonolysis products bring the products together in such a way that 'O' atoms are face to face, replace O by C=C bond.

26. Natural rubber is a linear polymers of isoprene and is also called as *cis*-1, 4-polyisoprene.

It is a linear 1, 4-polymer of isoprene (2-methyl-1,3-butadiene).



natural rubber

27. Orlon is an addition polymer.

$$n\,CH_2{=}CHCN \xrightarrow[\text{high pressure}]{\text{Polymerisation}} [CH_2{-}CH(CN)]_n$$

Orlon
polyacrylonitrile

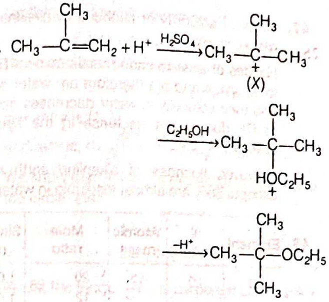
28.

A.	XeF_4	sp^3d^2
B.	H_2O	sp^3
C.	PCl_5	sp^3d
D.	$[Pt(NH_3)_4]^{2+}$	$d\,sp^2$

29. In acidic medium

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$$

$$\text{eq. wt} = \frac{\text{molecular weight}}{\text{no. of electrons transferred}} = \frac{M}{6}$$

30. 

31. A new carbon- carbon bond formation takes place in Friedel- Craft reaction and Reimer-Tiemann reaction.

32. $\because$ $\pi = \dfrac{n}{V}RT$ $\therefore$ $\pi \propto n$

n (glucose) $= \dfrac{10}{180} = 0.055$

n (urea) $= \dfrac{10}{60} = 0.16$

n (sucrose) $= \dfrac{10}{342} = 0.029$

$\underset{\pi_2}{\pi\,(\text{urea})} > \underset{\pi_1}{\pi\,(\text{glucose})} > \underset{\pi_3}{\pi\,(\text{sucrose})}$

33. $C_6H_5CHO + (CH_3CO)_2O \xrightarrow[453\,K]{CH_3COONa}$

$C_6H_5CH{=}CH\overset{O}{\overset{\|}{C}}{-}O{-}\overset{O}{\overset{\|}{C}}{-}CH_3 \xrightarrow[-CH_3COOH]{H_2O/H^+}$

$C_6H_5CH{=}CHCOOH$
Cinnamic acid

34. ${}^{235}_{92}U \xrightarrow{-\alpha} {}^{231}_{90}Th$

The elements ${}_{89}Ac$ to ${}_{103}Lr$ belong to IIIB group of periodic table (Actinoid series).

35. $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2m \times (KE)}}$

$\lambda_1 = \dfrac{h}{\sqrt{2m \times 200}}$

$\lambda_2 = \dfrac{h}{\sqrt{2m \times 50}}$

$\dfrac{\lambda_1}{\lambda_2} = \dfrac{1 \times \sqrt{50}}{\sqrt{200} \times 1} = \dfrac{1}{2}$

$\lambda_1 : \lambda_2 = 1:2$

36. Fe, Co, Ni, Cu have almost same atomic radii [126, 125, 125, 128 (in pm) respectively]. This is due to the fact that successive addition of d- electrons screen the outer electrons (4s-) from invard pull of the nucleus. As a result of this, the size of the atom does not alter much in moving from chromium to copper.

37. $CH_3CH_2C{\equiv}CH \underset{B}{\overset{A}{\rightleftharpoons}} CH_3{-}C{\equiv}C{-}CH_3$

$\underset{\text{but-1-yne}}{CH_3CH_2C{\equiv}CH} \xrightarrow{\text{Ethanolic KOH}}$

$\underset{\text{Intermediate}}{[CH_3CH{=}C{=}CH_2]} \longrightarrow \underset{\text{but-2-yne}}{CH_3C{\equiv}CCH_3}$

Acetylene homologues isomerise when heated with ethanolic KOH, the triple bond moving towards the centre of the chain.

$CH_3C{\equiv}CCH_3 \xrightarrow[\text{(ii) } H^+]{\text{(i) NaNH}_2\text{, paraffin}} CH_3{-}CH_2C{\equiv}CH$

When alkynes are heated with sodamide in an inert solvent (paraffin), triple bond shifts towards the end of the chain.

38. $CuCO_3 \cdot Cu(OH)_2 \longrightarrow \underset{(A)}{Cu_2O} + CO_2 + H_2O$

$Cu_2O + C \longrightarrow \underset{(B)}{2Cu} + CO$

39.

	Formula	Name of the compound
A.	$NaNO_3$	Chile salt petre
B.	$Na(NH_4)HPO_4$	Microcosmic salt
C.	$NaHCO_3$	Baking soda
D.	$Na_2CO_3 \cdot 10H_2O$	Washing soda

40. $2MnO_2 + 4HNO_3 \rightarrow 2Mn(NO_3)_2 + 2H_2O + O_2$

$2Mn(NO_3)_2 + 5PbO_2 + 6HNO_3 \longrightarrow$

$\underset{\text{Permanganic acid (violet pink)}}{2HMnO_4} + 5Pb(NO_3)_2 + 2H_2O$

41. ${}^{27}_{13}Al + {}^{4}_{2}He \longrightarrow {}^{30}_{14}Si + {}^{1}_{1}H$

Bombardment of stable elements with high energy α-particles, protons, neutrons, deutrons or γ-rays produce radioactive nuclides.

These radio-nuclides do not occur naturally and may be called man-made or artificial nuclides. The radioactivity exhibited by these artificial radio nuclides is referred to as artificial radioactivity.

42. Edge length of (NaCl crystal) unit cell

$$= 2(r_{Na^+} + r_{Br^-})$$

43. Magnetic moment $\propto$ no. of unpaired electron.

$Zn^{2+} = 3d^{10}4s^0$ unpaired electron = 0;

$\mu = 0$

$Ti^{3+} = 3d^{10}4s^0$ unpaired electron = 1

$\mu = \sqrt{n(n+2)}$

$= 1.73$

$Sc^{3+} = 3d^0 4s^0$ unpaired electron = 0;

$\mu = 0$

$Mn^{2+} = 3d^5 4s^0$ unpaired electrons = 5

$\mu = \sqrt{5(5+2)}$

$= 5.91$

44. Elektron is an alloy of different metals such as aluminium, zinc, silver, maganese, yttrium, neodymium, gadolinium, zirconium and other rare earth metals.

45. Edge length = 400 pm

$$= 2(r^+ + r^-)$$
$$= 2r^+ + 2r^-$$
$$400 = 2 \times 75 + 2r^-$$

or $\quad r^- = 125\,\text{pm}$

46. In lead, +2 oxidation state becomes more stable due to inert pair effect.

This is also supported by the fact that the Pb(+4) compounds act as strong oxidising agent.

47. Order of solubility of fluorides of alkaline earth metals in water is

Halides of alkaline earth metals (except Be) are ionic solids and are therefore are water soluble and their solubility in water decreases from Mg to Ba due to the decrease in the hydration energy.

However, fluorides of alkanline earth metals excepts BeF_2 are almost insoluble in water.

48.

Element	% amount	Atomic mass	Mole ratio	Simplest ratio
C	20	12	$\frac{20}{12} = 1.66$	$\frac{1.66}{1.66} = 1$
N	46.66	14	$\frac{46.66}{14} = 3.33$	$\frac{3.33}{1.66} = 2$
H	6.66	1	$\frac{6.66}{1} = 6.66$	$\frac{6.66}{1.66} = 4$
O	26.68	16	$\frac{26.68}{16} = 1.66$	$\frac{1.66}{1.66} = 1$

$\therefore$ Empirical formula $= CN_2H_4O$

49. Metals having negative values of $E°$(reduction) will displace H_2 with acids.

50. $KI + I_2 \longrightarrow KI_3$ or $I^- + I_2 \rightarrow I^-_3$
from KI

On increasing concentration of KI and I_2 two fold and three fold respectively, the concentration of KI_3 becomes two fold.

Mathematics

1. Given equations of plane are

$2x - y + z = 6$ and $x + y + 2z = 3$

On comparing with $ax + by + cz = d$, we get

$a_1 = 2, b_1 = -1, c_1 = 1$

and $\quad a_2 = 1, b_2 = 1, c_2 = 2$

$\therefore$ Angle between two planes is

$$\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 \times 1 + (-1) \times 1 + 1 \times 2}{\sqrt{2^2 + (-1)^2 + (1)^2}\sqrt{1^2 + 1^2 + (2)^2}}\right)$$

$$= \cos^{-1}\left(\frac{2 - 1 + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}\right)$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{6} \times \sqrt{6}}\right) = \cos^{-1}\left(\frac{3}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right)$$

$= 60°$

2. Let the equation of tangent be

$$y = mx + c \quad \ldots(i)$$

Given equation of circle is

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

Its centre is (1, 1)

and radius, $r = \sqrt{1^2 + 1^2 - 1} = \sqrt{1 + 1 - 1} = 1$

We know that, if line $y = mx + c$ be a tangent to the circle, then

$$c = \pm r\sqrt{1 + m^2}$$

$\therefore \quad c = \pm 1\sqrt{1 + m^2} \quad \ldots(ii)$

Since, the tangent line is perpendicular to $y = x$.

$\therefore \quad m \times 1 = -1 \quad (\because m_1m_2 = -1)$

$\Rightarrow \quad m = -1$

On putting $m = -1$ in Eq. (ii), we get

$$c = \pm 1\sqrt{1 + (-1)^2}$$
$$= \pm\sqrt{1 + 1}$$
$$= \pm\sqrt{2}$$

On putting the values of $m = -1$ and $c = \pm\sqrt{2}$ in Eq. (i), we get

$$y = -x \pm \sqrt{2}$$

3. Given, $P(A \cup B) = P(A \cap B)$

$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = 2P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = 2 \times P(A)\,P(B/A)$

$$\left[\because P(B/A) = \frac{P(A \cap B)}{P(A)}\right]$$

4. Suppose P divides the line joining points $A(7, 2, 1)$ and $B(10, 5, 7)$ in the ratio $\lambda : 1$. Then,

$\lambda \quad P \quad 1$

A(7,2,1) ——————— B(10,5,7)

Coordinates of P are

$$\left(\frac{10\lambda + 7}{\lambda + 1}, \frac{5\lambda + 2}{\lambda + 1}, \frac{7\lambda + 1}{\lambda + 1}\right)$$

Since, y-coordinate of P is 4.

$\therefore \quad \frac{5\lambda + 2}{\lambda + 1} = 4$

$\Rightarrow \quad 5\lambda + 2 = 4\lambda + 4$

$\Rightarrow \quad \lambda = 2$

$\therefore$ x-coordinate of $P = \left(\frac{10 \times 2 + 7}{2 + 1}\right)$

$$= \left(\frac{20 + 7}{3}\right) = \frac{27}{3}$$

$\Rightarrow \quad x = 9$

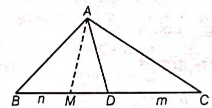
and z-coordinate of $P = \left(\frac{7 \times 2 + 1}{2 + 1}\right)$

$$= \left(\frac{14 + 1}{3}\right) = \frac{15}{3}$$

$\Rightarrow \quad z = 5$

5. $mBD^2 + nCD^2 + (m + n)AD^2$

$$= mBD^2 + nCD^2 + mAD^2 + nAD^2$$



$= m(BD^2 + AD^2) + n(CD^2 + AD^2)$

$= m(AB^2 + 2DM \cdot DB) + n(AC^2 - 2DM \cdot DC)$

$= mAB^2 + m \cdot 2DM \cdot DB + nAC^2 - n \cdot 2DM \cdot DC$

$= mAB^2 + nAC^2 + 2DM(mDB - nDC)$

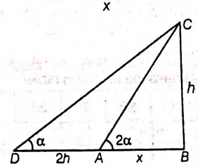
$= mAB^2 + nAC^2 + 2DM(mn - nm)$

$= mAB^2 + nAC^2$

6. Let the height of the pole be $BC = h$ m.

In ΔABC,

$$\tan 2\alpha = \frac{h}{x} \quad \ldots(i)$$



and in ΔDBC,

$$\tan\alpha = \frac{h}{2h + x}$$

$$\Rightarrow \quad \tan\alpha = \frac{\frac{h}{x}}{2\left(\frac{h}{x}\right)+1}$$

$$\Rightarrow \quad \tan\alpha = \frac{\tan 2\alpha}{2\tan 2\alpha + 1}$$

$$\Rightarrow \quad \tan\alpha\,(2\tan 2\alpha + 1) = \tan 2\alpha$$

$$\Rightarrow \tan\alpha\left[2\times\frac{2\tan\alpha}{1-\tan^2\alpha}+1\right] = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\Rightarrow \quad [4\tan\alpha + 1 - \tan^2\alpha] = 2$$

$$\Rightarrow \quad \tan^2\alpha - 4\tan\alpha + 1 = 0$$

$$\Rightarrow \quad \tan\alpha = \frac{+4\pm\sqrt{16-4}}{2\times 1}$$

$$= \frac{4\pm\sqrt{12}}{2} = \frac{4\pm 2\sqrt{3}}{2}$$

$$\Rightarrow \quad \tan\alpha = 2\pm\sqrt{3}$$

Taking '−' sign, we get

$$\Rightarrow \quad \tan\alpha = 2-\sqrt{3}$$

$$\Rightarrow \quad \tan\alpha = \tan 15^\circ$$

$$\Rightarrow \quad \alpha = 15^\circ \text{ or } \frac{\pi}{12}$$

7. Given, $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]} & \text{for}[x]\neq 0 \\ 0 & \text{for}[x]=0 \end{cases}$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{\sin(1+[x])}{[x]}$$

$$= \frac{\sin[1-1]}{-1} = \frac{\sin 0}{-1} = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{\sin(1+[x])}{[x]}$$

$$= \frac{\sin(1+0)}{0} = \infty$$

Hence, limit does not exist.

8. Given equation of straight lines are

$$y = (2-\sqrt{3})x + 5$$

and $\quad y = (2+\sqrt{3})x - 7$

On comparing with $y = mx + c$, we get

$$m_1 = 2-\sqrt{3}$$

and $\quad m_2 = 2+\sqrt{3}$

$$\therefore \quad \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{2+\sqrt{3}-(2-\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{2\sqrt{3}}{1+(4-3)} = \frac{2\sqrt{3}}{1+4-3}$$

$$\Rightarrow \quad \tan\theta = \frac{2\sqrt{3}}{2}$$

$$= \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

9. NAND is a universal gate, because with the help of this gate, any logic gate can be designed.

10. Let $I = \int_1^7 \frac{dx}{x}$

$$\because \quad h = \frac{b-a}{n}$$

Let $\quad n = 12 \quad \therefore \quad h = \frac{7-1}{12} = \frac{6}{12}$

$$= 0.5$$

Then, we have the following table

x	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
$\frac{1}{x}$	1	0.67	0.5	0.4	0.33	0.29	0.25	0.22	0.2	0.18	0.17	0.15	0.14
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}

Now, by Simpson's rule,

$$\int_1^7 \frac{dx}{x} = \frac{h}{3}[(y_0 + y_{12}) + 4(y_1 + y_3 + y_5 + y_7 + y_9 + y_{11}) + 2(y_2 + y_4 + y_6 + y_8 + y_{10})$$

$$= \frac{0.5}{3}[(1+0.14) + 4(0.67 + 0.4 + 0.29 + 0.22 + 0.18 + 0.15) + 2(0.5 + 0.33 + 0.25 + 0.2 + 0.17)]$$

$$= \frac{5}{30}[1.14 + 4\times 1.91 + 2\times 1.45]$$

$$= \frac{5}{30}[1.14 + 7.64 + 2.9] = \frac{5\times 11.68}{30} = 1.958$$

11. Equation of plane passing through $(-1, 1, 1)$ is

$$a(x+1) + b(y-1) + c(z-1) = 0 \quad \ldots\text{(i)}$$

Also, it is passing through $(1, -1, 1)$.

$$\therefore \quad a(1+1) + b(-1-1) + c(1-1) = 0$$

$$\Rightarrow \quad 2a - 2b + 0c = 0 \quad \ldots\text{(ii)}$$

Also, required equation of plane (i) is perpendicular to $x + 2y + 2z = 5$.

$$\therefore \quad a\times 1 + b\times 2 + c\times 2 = 0$$

$$\Rightarrow \quad a + 2b + 2c = 0 \quad \ldots\text{(iii)}$$

Eqs. (ii) and (iii) are identical.

$$\therefore \quad \frac{a}{-4-0} = \frac{-b}{4-0} = \frac{c}{4+2}$$

$$\Rightarrow \quad \frac{a}{-4} = \frac{b}{-4} = \frac{c}{6}$$

$$\Rightarrow \quad \frac{a}{-2} = \frac{b}{-2} = \frac{c}{3} = \lambda \quad \text{(say)}$$

$$\Rightarrow a = -2\lambda, b = -2\lambda, c = 3\lambda$$

On putting the values of a, b and c in Eq. (i), we get

$$-2\lambda(x+1) - 2\lambda(y-1) + 3\lambda(z-1) = 0$$

$$\Rightarrow \quad \lambda[-2x - 2 - 2y + 2 + 3z - 3] = 0$$

$$\Rightarrow \quad -2x - 2y + 3z - 3 = 0$$

$$\Rightarrow \quad 2x + 2y - 3z + 3 = 0$$

12. $\sin 50^\circ\cos 10^\circ + \cos 50^\circ \sin 10^\circ$

$$= \sin(50^\circ + 10^\circ)$$

$$= \sin 60^\circ = \frac{\sqrt{3}}{2}$$

13. Given, $g(x) = f(\tan^2 x - 2\tan x + 4)$

On differentiating w.r.t. x, we get

$$g'(x) = f'(\tan^2 x - 2\tan x + 4) \times (2\tan x\sec^2 x - 2\sec^2 x)$$

$$= f'(\tan^2 x - 2\tan x + 4)\,2\sec^2 x(\tan x - 1)$$

$$\because \quad f'(x) > 0\ \forall\ x\in\left(0, \frac{\pi}{2}\right)$$

Also, $\quad 2\sec^2 x > 0\ \forall\ x\in\left(0, \frac{\pi}{2}\right)$

$$\left[\because x\in\left(0, \frac{\pi}{2}\right), \text{given}\right]$$

But $\tan x - 1 > 0\ \forall\ x\in\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$$\therefore \quad g'(x) > 0\ \forall\ x\in\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

Hence, $g(x)$ is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

14. The total number of sample points in a sample space of single throw of two dice, $n(S) = 36$

Let E_1 = Event of getting a sum 7

E_2 = Event of getting a sum 9

$$\therefore \quad E_1 = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}$$

$$\Rightarrow \quad n(E_1) = 6$$

and $\quad E_2 = \{(3,6), (6,3), (4,5), (5,4)\}$

$$\Rightarrow \quad n(E_2) = 4$$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

and $\quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Now, $P(E_1\cup E_2) = P(E_1) + P(E_2)$

$$= \frac{1}{6} + \frac{1}{9} = \frac{6+4}{36} = \frac{10}{36} = \frac{5}{18}$$

15. Compiler is a system software which transforms source code written in a programming language into another computer language.

16. Given, $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \left(\frac{3x+x^3}{1+3x^2}\right)$

$$\therefore \quad fog(x) = f\{g(x)\} = f\left(\frac{3x+x^3}{1+3x^2}\right)$$

$$= \log\left[\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right]$$

$$= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$

$$= \log\left[\frac{(1+x)^3}{(1-x)^3}\right] = \log\left(\frac{1+x}{1-x}\right)^3$$

$$= 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$$

17. Given, $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$

Using L' Hospital's rule, we get

$$\lim_{t\to x}\frac{t^2 f'(x) - 2xf(t)}{-1} = 1$$

$$\Rightarrow \quad x^2 f'(x) - 2x f(x) + 1 = 0$$

$$\Rightarrow \quad \frac{x^2 f'(x) - 2xf(x)}{(x^2)^2} + \frac{1}{x^4} = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = -\frac{1}{x^4}$$

On integrating both sides, we get

$$\frac{f(x)}{x^2} = +\frac{1}{3x^3} + c \Rightarrow f(x) = \frac{1}{3x} + cx^2$$

Also, $f(1)=1 \Rightarrow 1 = \frac{1}{3\times 1} + c(1)^2$

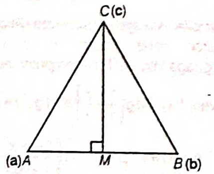
$$\Rightarrow \quad 1 = \frac{1}{3} + c \Rightarrow \frac{2}{3} = c$$

$$\therefore \quad f(x) = \frac{1}{3x} + \frac{2}{3x^2}$$

18. Given, position vectors of three non-collinear points A, B, C are **a**, **b** and **c**.

Let CM be the perpendicular from C on AB.

Then,



Area of $\Delta ABC = \frac{1}{2} AB\cdot CM = \frac{1}{2}|AB\cdot CM|$

But area of $\Delta ABC = \frac{1}{2}|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}|$

$$\therefore \quad \frac{1}{2}|AB\cdot CM| = \frac{1}{2}|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}|$$

$$\Rightarrow \quad CM = \frac{|\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}|}{|\mathbf{b}-\mathbf{a}|}$$

19. We have,

$$\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx = A\sin^{-1}x + Bx\sqrt{1-x^2} + C$$

Let $I = \int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

Put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta\, d\theta$ $\therefore$

$$I = -\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}2\sin 2\theta\, d\theta$$

$$= -\int \sin\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\}2\sin 2\theta\, d\theta$$

$$= -\int \sin\{2\tan^{-1}\sqrt{\tan^2\theta}\}2\sin 2\theta\, d\theta$$

$$= -\int \sin\{2\tan^{-1}\tan\theta\}2\sin 2\theta\, d\theta$$

$$= -\int [\sin(2\theta)]2\sin 2\theta\, d\theta$$

$$= -\int 2\sin^2 2\theta\, d\theta$$

$$= -\int (1-\cos 4\theta)\, d\theta$$

$$= -\left[\theta - \frac{\sin 4\theta}{4}\right] + C$$

$$= \left[-\frac{1}{2}\cos^{-1}x + \frac{1}{4}\times 2\sin 2\theta\cos 2\theta\right] + C$$

$$= \left[-\frac{1}{2}\left(\frac{\pi}{2} - \sin^{-1}x\right) + \frac{1}{2}\sqrt{1-x^2}\times x\right] + C$$

$$= \frac{1}{2}\sin^{-1}x + \frac{x}{2}\sqrt{1-x^2} + \left(C - \frac{\pi}{4}\right)$$

But $I = A\sin^{-1}x + Bx\sqrt{1-x^2} + C$

$$\therefore \quad A = \frac{1}{2} \text{ and } B = \frac{1}{2}$$

Hence, $A + B = \frac{1}{2} + \frac{1}{2} = 1$

20. Let $E = (1 + x + x^3 + x^4)^{10}$

$$= [1 + x + x^3(1+x)]^{10} = (1+x)^{10}(1+x^3)^{10}$$

$$= [{}^{10}C_0 + {}^{10}C_1x^1 + {}^{10}C_2x^2 + \ldots + {}^{10}C_9x^9 + {}^{10}C_{10}x^{10}]$$

$$\times [{}^{10}C_0 1 + {}^{10}C_1 \times (x^3) + {}^{10}C_2(x^3)^2 + {}^{10}C_3(x^3)^3 + \ldots]$$

$\therefore$ Coefficient of x^4 in $E = {}^{10}C_4 \times {}^{10}C_0 + {}^{10}C_1 \times {}^{10}C_1$

$$= \frac{10!}{6!\times 4!}\times 1 + 10\times 10$$

$$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} + 100$$

$$= 210 + 100$$

$$= 310$$

21. Given, $A = \begin{bmatrix}4 & 11\\ 2 & 6\end{bmatrix}$

Now, $|A| = \begin{vmatrix}4 & 11\\ 2 & 6\end{vmatrix}$

$$= 24 - 22 = 2$$

$$\text{adj } A = \begin{bmatrix}6 & -11\\ -2 & 4\end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{2}\begin{bmatrix}6 & -11\\ -2 & 4\end{bmatrix}$$

$$= \begin{bmatrix}3 & \frac{-11}{2}\\ -1 & 2\end{bmatrix}$$

22. Let equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Given equation of circle is

$$x^2 + y^2 - 4x + 8 = 0$$

The centres of above circles are $(-g, -f)$ and $(2, 0)$.

Condition of orthogonality is

$$2(g_1g_2 + f_1f_2) = c_1 + c_2$$

$$\therefore \quad 2(g\times(-2) + (f)\times 0) = c + 8$$

$$\Rightarrow \quad -4g = c + 8 \quad \ldots\text{(i)}$$

Also, the assume circle touch the line $x + 1 = 0$.

$\therefore$ The perpendicular drawn from centre to the line is equal to radius.

$$\therefore \quad \frac{-g+1}{\sqrt{1^2}} = \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \quad -g + 1 = \sqrt{g^2 + f^2 - c}$$

On squaring both sides, we get

$$g^2 + 1 - 2g = g^2 + f^2 - c$$

$$\Rightarrow \quad c = f^2 + 2g - 1$$

Putting the value of c in Eq. (i), we get

$$-4g = f^2 + 2g - 1 + 8$$

$$\Rightarrow \quad f^2 + 2g + 4g + 7 = 0$$

$$\Rightarrow \quad f^2 + 6g + 7 = 0$$

$\therefore$ Locus of centre of circle is $y^2 + 6x + 7 = 0$.

23. Given,

$$f(x) = \begin{vmatrix}\sin 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2\\ \cos 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right)\\ \tan 3x & 4 & 1 + 2\tan 3x\end{vmatrix}$$

On differentiating w.r.t. x, we get

$$f'(x) = \begin{vmatrix}\frac{d}{dx}(\sin 3x) & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2\\ \frac{d}{dx}(\cos 3x) & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right)\\ \frac{d}{dx}(\tan 3x) & 4 & 1 + 2\tan 3x\end{vmatrix}$$

$$+ \begin{vmatrix}\sin 3x & \frac{d}{dx}(1) & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2\\ \cos 3x & \frac{d}{dx}(-1) & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right)\\ \tan 3x & \frac{d}{dx}(4) & 1 + 2\tan 3x\end{vmatrix}$$

$$+ \begin{vmatrix}\sin 3x & 1 & 2\frac{d}{dx}\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2\\ \cos 3x & -1 & 2\frac{d}{dx}\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right)\\ \tan 3x & 4 & \frac{d}{dx}(1 + 2\tan 3x)\end{vmatrix}$$

$$= \begin{vmatrix}3\cos 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2\\ 3\sin 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right)\\ 3\sec^2 3x & 4 & 1 + 2\tan 3x\end{vmatrix}$$

$$+ \begin{vmatrix}\sin 3x & 0 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2\\ \cos 3x & 0 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right)\\ \tan 3x & 0 & 1 + 2\tan 3x\end{vmatrix}$$

$$+\begin{vmatrix} \sin 3x & 1 & 2\times 2\left(\cos\frac{3x}{2}+\sin\frac{3x}{2}\right)\times\left(\frac{-3}{2}\sin\frac{3x}{2}+\frac{3}{2}\cos\frac{3x}{2}\right) \\ \cos 3x & -1 & 2\left(-2\cos\frac{3x}{2}\times\frac{3}{2}\sin\frac{3x}{2}-2\sin\frac{3x}{2}\times\frac{3}{2}\cos\frac{3x}{2}\right) \\ \tan 3x & 4 & (0+2\times 3\sec^2 3x) \end{vmatrix}$$

At $x=(2n+1)\pi$,

$$f'(x)=\begin{vmatrix} 3(-1) & 1 & 2(1) \\ 0 & -1 & 2(-1) \\ 3 & 4 & 1+0 \end{vmatrix}+0+\begin{vmatrix} 0 & 1 & 4(0-1)\times\left[-\frac{3}{2}(-1)+\frac{3}{2}\times 0\right] \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} -3 & 1 & 2 \\ 0 & -1 & -2 \\ 3 & 4 & 1 \end{vmatrix}+0+\begin{vmatrix} 0 & 1 & -6 \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$=[-3(-1+8)-1(0+6)+2(0+3)]+[0-1(0-0)-6(-4)]$$

$$=-21+24=3$$

24. Given equation of line is $lx+my+n=0$

and equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$

$\therefore$ The equation of any normal to the ellipse is

$$5x\sec\theta-3y\,\text{cosec}\,\theta=25-9$$

$\left(\because \text{ the equation of any normal to the ellipse } \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } ax\sec\theta-by\,\text{cosec}\,\theta=a^2-b^2\right)$

$\Rightarrow \quad 5x\sec\theta-3y\,\text{cosec}\,\theta-16=0 \quad \ldots(i)$

As the Eq. (i) is the normal to the ellipse.

$$\therefore \quad \frac{5\sec\theta}{l}=\frac{-3\,\text{cosec}\,\theta}{m}=\frac{-16}{n}$$

$\Rightarrow \quad \cos\theta=\frac{5n}{-16l}$ and $\sin\theta=\frac{3n}{16m}$

$\because \quad \cos^2\theta+\sin^2\theta=1$

$$\therefore \quad \left(\frac{5n}{16l}\right)^2+\left(\frac{3n}{16m}\right)^2=1$$

$$\Rightarrow \quad \frac{25n^2}{256l^2}+\frac{9n^2}{256m^2}=1$$

$$\Rightarrow \quad \frac{25}{l^2}+\frac{9}{m^2}=\frac{256}{n^2}$$

25. Given, $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$

$\Rightarrow \quad 4(1-\sin x)+\sin x=a\sin x(1-\sin x)$

$\Rightarrow \quad 4-4\sin x+\sin x=a\sin x-a\sin^2 x$

$\Rightarrow \quad a\sin^2 x-(3+a)\sin x+4=0 \quad \ldots(i)$

It is a quadratic equation in $\sin x$, so

$D\geq 0$

$\Rightarrow \quad (3+a)^2-4\times 4a\geq 0$

$\Rightarrow \quad 9+a^2+6a-16a\geq 0$

$\Rightarrow \quad a^2-10a+9\geq 0$

$\Rightarrow \quad (a-1)(a-9)\geq 0$

$\Rightarrow \quad a\geq 9$ or $a\leq -9$

Now, at $a=9$, Eq. (i) becomes

$9\sin^2 x-12\sin x+4=0$

$\Rightarrow \quad (3\sin x-2)^2=0$

$\Rightarrow \quad \sin x=\frac{2}{3}<0$

$\Rightarrow \quad x\in\left(0,\frac{\pi}{2}\right)$

Hence, least value of a is 9.

26. If one line of regression coefficient is less than unity, then other will be greater than unity.

27. Given edges of a parallelopiped are

$\mathbf{a}=2\hat{i}-3\hat{j}+\hat{k}$, $\mathbf{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\mathbf{c}=2\hat{i}+\hat{j}-\hat{k}$

$\therefore$ Volume of parallelopiped $=[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]$

$$=\left|\begin{vmatrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}\right|$$

$=|2(1-2)+3(-1-4)+1(1+2)|$

$=|-2-15+3|=14$ cu units

28. Let $f(x)=x^3-6x+1$

Now, $f(2)=(2)^3-6\times 2+1$

$=8-12+1=-3$

and $f(3)=(3)^3-6\times 3+1$

$=27-18+1=10$

Here, we see that $f(2)$ and $f(3)$ have opposite signs, so one of the roots lies in (2, 3).

29. RAM is a volatile memory, because when computer is turned off, the content of RAM is immediately lost.

30. Given, $\lim_{n\to\infty}\sum\frac{\log(n+r)-\log n}{n}=2\left(\log 2-\frac{1}{2}\right)$

or $\lim_{n\to\infty}\sum\frac{1}{n}\log\left(1+\frac{r}{n}\right)=2\left(\log 2-\frac{1}{2}\right) \quad \ldots(i)$

Let $A=\lim_{n\to\infty}\frac{1}{n^\lambda}[(n+1)^\lambda(n+2)^\lambda\ldots(n+n)^\lambda]^{1/n}$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^\lambda\left(1+\frac{2}{n}\right)^\lambda\ldots\left(1+\frac{n}{n}\right)^\lambda\right]^{1/n}$$

On taking log both sides, we get

$$\log A=\lim_{n\to\infty}\frac{1}{n}\left[\log\left(1+\frac{1}{n}\right)^\lambda+\log\left(1+\frac{2}{n}\right)^\lambda+\ldots+\log\left(1+\frac{n}{n}\right)^\lambda\right]$$

$\Rightarrow \quad \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\lambda\log\left(1+\frac{r}{n}\right)$

$\Rightarrow \quad \log A=\lambda\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log\left(1+\frac{r}{n}\right)$

$\Rightarrow \quad \log A=2\lambda\left(\log 2-\frac{1}{2}\right) \quad$ [from Eq. (i)]

$\Rightarrow \quad \log A=\log 4^\lambda-\lambda$

$\Rightarrow \quad \log A=\log 4^\lambda-\lambda\log e$

$\Rightarrow \quad \log A=\log\frac{4^\lambda}{e^\lambda}$

$\therefore \quad A=\left(\frac{4}{e}\right)^\lambda$

31. Given, $\lim_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=-\frac{1}{12} \quad \ldots(i)$

$\Rightarrow \quad \frac{\sin\sin 0-\sin 0}{a(0)^3+b(0)^5+c}=-\frac{1}{12}$

$\Rightarrow \quad \frac{0}{c}=-\frac{1}{12} \Rightarrow c=0$

Applying L' Hospital's rule in Eq (i), we get

$$\lim_{x\to 0}\frac{\cos(\sin x)\cos x-\cos x}{3ax^2+5bx^4+0}=-\frac{1}{12}$$

$$\Rightarrow \quad \lim_{x\to 0}\frac{\cos x(\cos(\sin x)-1)}{x^2(3a+5bx)}=-\frac{1}{12}$$

$$\Rightarrow \quad \lim_{x\to 0}\cos x\,\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{x^2(3a+5bx)}=-\frac{1}{12}$$

$$\Rightarrow \quad \frac{\cos 0}{3a+5b\times 0}\times\lim_{x\to 0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{4\left(\frac{\sin x}{2}\right)^2\times\left(\frac{x}{\sin x}\right)^2}=-\frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{3a+0}\times\frac{1}{2}\times\frac{1}{1}=-\frac{1}{12} \quad \left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right)$$

$\Rightarrow \quad \frac{1}{6a}=-\frac{1}{12} \Rightarrow 6a=-12$

$\therefore \quad a=-2$

Hence, $a=-2$, $b\in R$ and $c=0$

32. Let $x=\sqrt{12}$

$\Rightarrow \quad x^2=12$

$\Rightarrow \quad x^2-12=0$

Let $\quad f(x)=x^2-12$

The first approximation in the Newton-Raphson method is given by

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$=x_0-\frac{x_0^2-12}{2x_0}$$

$$=\frac{x_0^2+12}{2x_0}$$

$\because \quad 3<\sqrt{12}<3.5$

We can take $x_0=3.5$

$$\therefore \quad x_1=\frac{(3.5)^2+12}{2\times 3.5}=\frac{12.25+12}{7}$$

$$=\frac{24.25}{7}$$

$$=3.464$$

33. Given equation is

$$\frac{(3-i)^2}{2+i}=A+iB$$

$$\Rightarrow \quad \frac{9-1-6i}{2+i}=A+iB$$

$$\Rightarrow \quad \frac{8-6i}{2+i}=A+iB$$

$$\Rightarrow \frac{2(4-3i)}{2+i}\times\frac{2-i}{2-i}=A+iB$$

$$\Rightarrow \frac{2[8-4i-6i-3]}{4+1}=A+iB$$

$$\Rightarrow \frac{2[5-10i]}{5}=A+iB$$

$$\Rightarrow 2-4i=A+iB$$

On equating the real and imginary parts from both sides, we get

$$A=2$$

and $$B=-4$$

34. Given system of circles is

$$x^2+y^2+4x+7=0 \quad \ldots(i)$$

$$2(x^2+y^2)+3x+5y+9=0$$

or $$x^2+y^2+\frac{3}{2}x+\frac{5}{2}y+\frac{9}{2}=0 \quad \ldots(ii)$$

and $$x^2+y^2+y=0 \quad \ldots(iii)$$

The radical centre can be obtained by solving the Eqs. (i), (ii) and (iii).

On subtracting Eq. (ii) from Eq. (i), we get

$$4x-\frac{3}{2}x-\frac{5}{2}y+7-\frac{9}{2}=0$$

$$\Rightarrow \frac{5}{2}x-\frac{5}{2}y+\frac{5}{2}=0$$

$$\Rightarrow x-y+1=0 \quad \ldots(iv)$$

On subtracting Eq. (iii) from Eq. (i), we get

$$4x-y+7=0 \quad \ldots(v)$$

On solving Eqs. (iv) and (v), we get

$$x=-2$$

and $$y=-1$$

Hence, radical centre is $(-2,-1)$.

35. Let

$$S_n=1+5+12+22+35+\ldots+T_n$$

$$S_n=\quad 1+5+12+22+\ldots+T_n$$

$$0=1+4+7+10+13+\ldots-T_n$$

$$\Rightarrow T_n=1+4+7+10+13+\ldots+n \text{ terms}$$

This is an arithmetic series whose first term a is 1 and common difference d is 3.

$$\therefore T_n=\frac{n}{2}[2\times1+(n-1)3]$$

$$\Rightarrow T_n=\frac{n}{2}[2+3n-3]$$

$$=\frac{n}{2}[3n-1]$$

$$=\frac{1}{2}[3n^2-n]$$

$$\therefore S_n=\Sigma T_n$$

$$=\frac{1}{2}[\Sigma(3n^2-n)]$$

$$=\frac{1}{2}[3\Sigma n^2-\Sigma n]$$

$$=\frac{1}{2}\left[\frac{3n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}\right]$$

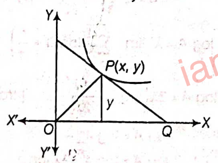
$$=\frac{n(n+1)}{2}\left[\frac{2n+1}{2}-\frac{1}{2}\right]$$

$$=\frac{n(n+1)}{2}\left[\frac{2n}{2}\right]=\frac{n^2(n+1)}{2}$$

36. Tangent drawn at any point (x, y) is

$$Y-y=\frac{dy}{dx}(X-x)$$

When $Y=0,\ X=x-y\dfrac{dx}{dy}$



$\because$ Area of $\Delta OPQ=a^2$ (given)

$$\therefore \left|\frac{1}{2}X\cdot y\right|=a^2$$

$$\Rightarrow \left|\left(x-y\frac{dx}{dy}\right)y\right|=2a^2$$

$$\Rightarrow xy-y^2\frac{dx}{dy}=\pm 2a^2$$

$$\Rightarrow \frac{dx}{dy}-\frac{x}{y}=\pm\frac{2a^2}{y^2}$$

Here, $P=-\dfrac{1}{y}$ and $Q=\pm\dfrac{2a^2}{y^2}$

$$\therefore \text{IF}=e^{\int P\,dy}=e^{\int -\frac{1}{y}dy}$$

$$=e^{-\log y}=e^{\log\frac{1}{y}}=\frac{1}{y}$$

Hence, required solution is

$$x\times\frac{1}{y}=\int\pm\frac{2a^2}{y^2}\times\frac{1}{y}dy$$

$$\Rightarrow \frac{x}{y}=\pm\frac{2a^2y^{-2}}{-2}+C$$

$$\Rightarrow x=Cy\pm\frac{a^2}{y}$$

which is the required curve.

37. Given, function $f[1,\infty)\to[1,\infty)$ is defined as $f(x)=2^{x(x-1)}$. It is an exponential function, so it is continuous and increasing in their domain. Thus, f^{-1} exists.

Let $$y=f(x)=2^{x(x-1)}$$

$$\Rightarrow \log y=x(x-1)\log 2$$

$$\Rightarrow (x^2-x)\log 2-\log y=0$$

$$\Rightarrow x^2-x-\frac{\log y}{\log 2}=0$$

$$\therefore x=\frac{+1\pm\sqrt{(-1)^2-4(1)\left(-\frac{\log y}{\log 2}\right)}}{2(1)}$$

$$=\frac{1\pm\sqrt{1+4\log_2 y}}{2}$$

Here, we see that range of $f(x)$ is $[1,\infty)$.

$$\because x=\frac{1+\sqrt{1+4\log_2 y}}{2}$$

$$\therefore f^{-1}(x)=\frac{1+\sqrt{1+4\log_2 y}}{2}$$

38. Given differential equation can be written as

$$\frac{dy}{dx}-\left(\frac{\tan 2x}{\cos^2 x}\right)y=\cos^2 x$$

Here, $$P=-\frac{\tan 2x}{\cos^2 x}=\frac{-\sin 2x}{\cos 2x\left(\frac{\cos 2x+1}{2}\right)}$$

and $$Q=\cos^2 x$$

$$\because \text{IF}=e^{\int P\,dx}=e^{-\int\frac{2\sin 2x}{\cos 2x(\cos 2x+1)}dx}$$

Put $\cos 2x=t\Rightarrow -2\sin 2x\,dx=dt$

$$\therefore \text{IF}=e^{\int\frac{1}{t}\left(\frac{1}{t+1}\right)dt}$$

$$=e^{\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt}$$

$$=e^{[\log t-\log(t+1)]}$$

$$=e^{\log\frac{t}{t+1}}$$

$$=e^{\log\frac{\cos 2x}{\cos 2x+1}}$$

$$=\frac{\cos 2x}{\cos 2x+1}$$

Now, solution is

$$y\times\frac{\cos 2x}{\cos 2x+1}=\int\frac{\cos 2x}{\cos 2x+1}\times\cos^2 x\,dx+C$$

$$=\int\frac{\cos 2x}{2\cos^2 x}\times\cos^2 x\,dx+C$$

$$=\frac{1}{2}\int\cos 2x\,dx+C$$

$$=\frac{1}{2}\times\frac{\sin 2x}{2}+C$$

$$\Rightarrow y\frac{\cos 2x}{\cos 2x+1}=\frac{1}{4}\sin 2x+C \quad \ldots(i)$$

But $$y\left(\frac{\pi}{6}\right)=\frac{3\sqrt{3}}{8}$$

$$\therefore \frac{3\sqrt{3}}{8}\times\frac{\cos\left(2\times\frac{\pi}{6}\right)}{\cos\left(2\times\frac{\pi}{6}\right)+1}=\frac{1}{4}\sin 2\left(\frac{\pi}{6}\right)+C$$

$$\Rightarrow \frac{\frac{3\sqrt{3}}{8}\times\frac{1}{2}}{\frac{1}{2}+1}=\frac{1}{4}\times\frac{\sqrt{3}}{2}+C$$

$$\Rightarrow \frac{3\sqrt{3}}{2\times8\times\frac{3}{2}}=\frac{\sqrt{3}}{8}+C$$

$$\Rightarrow \frac{\sqrt{3}}{8}=\frac{\sqrt{3}}{8}+C \Rightarrow C=0$$

From Eq. (i), we get

$$y\frac{\cos 2x}{\cos 2x+1}=\frac{1}{4}\sin 2x+0$$

$$\Rightarrow y=\frac{1}{4}\frac{\sin 2x}{\frac{\cos 2x}{\cos 2x+1}}$$

$$=\frac{1}{4}\frac{\sin 2x}{\frac{\cos^2 x-\sin^2 x}{2\cos^2 x}}$$

$$=\frac{1}{2}\cdot\frac{\sin 2x}{1-\tan^2 x}$$

39. Let the line of regression of y on x be

$$3x + 12y = 19$$

or $$y = \frac{-3x + 19}{12} \quad \dots(i)$$

and x on y be

$$x = \frac{-3y + 46}{9} \quad \dots(ii)$$

$\therefore$ Regression coefficient of y on x is

$$b_{yx} = \frac{-3}{12} = -\frac{1}{4}$$

and regression coefficient of x on y is

$$b_{xy} = -\frac{3}{9} = -\frac{1}{3}$$

$\therefore$ Correlation $r_{xy} = -\sqrt{b_{yx} \times b_{xy}}$

(here, we take negative sign outside the square root, because both regression coefficients are negative)

$$= -\sqrt{-\frac{1}{4} \times -\frac{1}{3}} = -\sqrt{\frac{1}{12}} = -\sqrt{0.083} = -0.289$$

40. $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$

$$= (1 + \omega^2 - \omega)(1 + \omega - \omega^2)$$
$$= (-\omega - \omega)(-\omega^2 - \omega^2) \quad (\because 1 + \omega + \omega^2 = 0)$$
$$= (-2\omega)(-2\omega^2)$$
$$= 4(\omega^3) = 4 \times 1 \quad (\because \omega^3 = 1)$$
$$= 4$$

41. Given, $\frac{dy}{dx} = \frac{y-1}{x^2 + x}$

$$\Rightarrow \frac{1}{y-1} dy = \frac{dx}{x(x+1)}$$

On integrating both sides, we get

$$\int \frac{1}{(y-1)} dy = \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx$$
$$\Rightarrow \log|y - 1| = [\log|x| - \log|x+1|] + C$$
$$\Rightarrow \log|y-1| = \log\left|\frac{x}{x+1}\right| + C \quad \dots(i)$$

At point (1, 0), we get

$$\log|0 - 1| = \log\left(\frac{1}{2}\right) + C$$
$$\Rightarrow C = -\log\frac{1}{2} + 0$$

From Eq. (i), we get

$$\log|y-1| = \log\left|\frac{x}{x+1}\right| - \log\frac{1}{2}$$
$$\Rightarrow \log(y-1) = \log 2 \left|\frac{x}{x+1}\right|$$
$$\Rightarrow (y-1) = \frac{2x}{x+1}$$
$$\Rightarrow (y-1)(x+1) = 2x$$
$$\Rightarrow (y-1)(x+1) - 2x = 0$$

42. Given, $f(x) = [\sin x + \cos x]$

$$= \left[\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)\right]$$
$$= \left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$$

We know that, greatest integer function is discontinuous on integer values.

Function $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ will gives integer values at $x = 90°, 135°, 180°, 270°, 315°$,

Hence, there are five points in the given interval, in which $f(x)$ is not continuous.

43. Let $I = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right)$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\left(\frac{\sin\frac{x}{2} + \cos\frac{x}{2} + \cos\frac{x}{2} - \sin\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2} - \cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$

44. Given vertices of a ΔABC are $A(-1, 3, 2)$, $B(2, 3, 5)$ and $C(3, 5, -2)$.

Now DR's of $AB = (2 + 1, 3 - 3, 5 - 2)$
$= (3, 0, 3)$

DR's of $BC = (3 - 2, 5 - 3, -2 - 5)$
$= (1, 2, -7)$

and DR's of $CA = (-1 - 3, 3 - 5, 2 + 2)$
$= (-4, -2, 4)$

Now, the angle between AB and BC,

$$\cos B = \frac{|3 \times 1 + 0 \times 2 + 3 \times (-7)|}{\sqrt{3^2 + 0^2 + 3^2}\sqrt{1^2 + 2^2 + (-7)^2}}$$
$$= \frac{|3 + 0 - 21|}{\sqrt{9 + 0 + 9}\sqrt{1 + 4 + 49}}$$
$$= \frac{18}{3\sqrt{2} \times 3\sqrt{6}}$$
$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

angle between BC and CA,

$$\cos C = \frac{|1 \times (-4) + 2(-2) + (-7)(4)|}{\sqrt{1^2 + 2^2 + (-7)^2}\sqrt{(-4)^2 + (-2)^2 + (4)^2}}$$
$$= \frac{|-4 - 4 - 28|}{\sqrt{1 + 4 + 49}\sqrt{16 + 4 + 16}}$$
$$= \frac{36}{\sqrt{54}\sqrt{36}} = \frac{36}{3\sqrt{6} \times 6}$$
$$= \frac{2}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

and angle between AC and AB,

$$\cos A = \frac{|-4 \times 3 + (-2) \times 0 + 4 \times 3|}{\sqrt{(-4)^2 + (-2)^2 + (4)^2}\sqrt{3^2 + 0^2 + 3^2}}$$
$$= |0|$$
$$\Rightarrow A = 90°$$

45. Let $I = \int_{1/e}^{1} \log x \, dx$

Here, we see that $\log x$ is negative for $x \in \left(1, \frac{1}{e}\right)$.

$$\therefore I = -\int_{1/e}^{1} \underset{II}{1} \times \underset{I}{(\log x)} dx$$
$$= -\left[\log x \times x - \int \frac{1}{x} \times x dx\right]_{1/e}^{1}$$
$$= -[x \log x - x]_{1/e}^{1}$$
$$= -\left[1 \log 1 - 1 - \left(\frac{1}{e}\log\frac{1}{e} - \frac{1}{e}\right)\right]$$
$$= -\left[0 - 1 - \left\{\frac{1}{e}(\log 1 - \log e)\right\} - \frac{1}{e}\right]$$
$$= -\left[-1 - \left\{\frac{1}{e}(0 - 1) - \frac{1}{e}\right\}\right]$$
$$= 1 + \left(-\frac{1}{e} - \frac{1}{e}\right) = 1 - \frac{2}{e}$$

46. $\lim_{x \to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3}$ $\left(\text{form } \frac{0}{0}\right)$

$$= \lim_{x \to 0} \frac{\sin\sqrt{x^2} \times 2x}{3x^2} \quad \text{(using L' Hospital's rule)}$$
$$= \lim_{x \to 0} \frac{2}{3}\frac{\sin x}{x} = \frac{2}{3} \times 1 = \frac{2}{3}$$

47. $[\mathbf{a} \times \mathbf{b} \;\; \mathbf{b} \times \mathbf{c} \;\; \mathbf{c} \times \mathbf{a}]$

$$= (\mathbf{a} \times \mathbf{b}) \cdot [(\mathbf{b} \times \mathbf{c}) \times (\mathbf{c} \times \mathbf{a})]$$
$$= (\mathbf{a} \times \mathbf{b}) \cdot [((\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a})\mathbf{c} - ((\mathbf{b} \times \mathbf{c}) \cdot \mathbf{c})\mathbf{a}]$$
$$= (\mathbf{a} \times \mathbf{b}) \cdot ([\mathbf{b}\;\mathbf{c}\;\mathbf{a}]\mathbf{c} - [\mathbf{b}\;\mathbf{c}\;\mathbf{c}]\mathbf{a})$$
$$= (\mathbf{a} \times \mathbf{b} \cdot \mathbf{c})[\mathbf{b}\;\mathbf{c}\;\mathbf{a}] - [\mathbf{a} \times \mathbf{b} \cdot \mathbf{a}]0$$
$$= [\mathbf{a}\;\mathbf{b}\;\mathbf{c}][\mathbf{a}\;\mathbf{b}\;\mathbf{c}] - 0$$
$$= [\mathbf{a}\;\mathbf{b}\;\mathbf{c}]^2$$

48. Geometric mean of a and $b = \sqrt{ab}$

$$\Rightarrow \sqrt{ab} = 16 \quad \text{(given)}$$
$$\Rightarrow ab = 256 \quad \dots(i)$$

And harmonic mean of a and $b = \frac{2ab}{a+b}$

$$\therefore \frac{2ab}{a+b} = \frac{64}{5}$$

(given)

$$\Rightarrow \frac{2 \times 256}{a+b} = \frac{64}{5} \quad \text{[from Eq. (i)]}$$
$$\Rightarrow a + b = 40 \quad \dots(ii)$$

Now, $$(a - b) = \sqrt{(a+b)^2 - 4ab}$$
$$= \sqrt{(40)^2 - 4 \times 256}$$
$$= \sqrt{1600 - 1024}$$
$$= \sqrt{576}$$
$$\Rightarrow a - b = 24 \quad \dots(iii)$$

On solving Eqs. (ii) and (iii), we get

$a = 32$ and $b = 8$

$$\therefore a : b = 32 : 8 = 4 : 1$$

49. Let $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

Applying $C_3 \to C_3 + C_2$, we get

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$= (a+b+c) \times 0$ ($\because C_1$ and C_3 are identical)

$= 0$

50. Let four terms in a GP be $ar^3, ar, \frac{a}{r}$ and $\frac{a}{r^3}$.

According to the given condition,

$$ar^3 + ar + \frac{a}{r} + \frac{a}{r^3} = 60 \quad \ldots(i)$$

and $$\frac{ar^3 + \frac{a}{r^3}}{2} = 18$$

$$\Rightarrow \quad ar^3 + \frac{a}{r^3} = 36 \quad \ldots(ii)$$

Now, from Eq. (i), we have

$$\left(ar + \frac{a}{r}\right) + ar^3 + \frac{a}{r^3} = 60$$

$$\Rightarrow \quad a\left(r + \frac{1}{r}\right) + 36 = 60 \quad \text{[from Eq. (ii)]}$$

$$\Rightarrow \quad a\left(r + \frac{1}{r}\right) = 24 \quad \ldots(iii)$$

On dividing Eq. (iii) by Eq. (ii), we get

$$\frac{a\left(r^3 + \frac{1}{r^3}\right)}{a\left(r + \frac{1}{r}\right)} = \frac{36}{24}$$

$$\Rightarrow \quad \frac{\left(r + \frac{1}{r}\right)\left(r^2 + \frac{1}{r^2} - 1\right)}{r + \frac{1}{r}} = \frac{3}{2}$$

$$\Rightarrow \quad 2\left(r^2 + \frac{1}{r^2} - 1\right) = 3$$

$$\Rightarrow \quad \frac{2(r^4 + 1 - r^2)}{r^2} = 3$$

$$\Rightarrow \quad 2r^4 + 2 - 2r^2 = 3r^2$$

$$\Rightarrow \quad 2r^4 - 5r^2 + 2 = 0$$

$$\Rightarrow \quad 2r^4 - 4r^2 - r^2 + 2 = 0$$

$$\Rightarrow \quad 2r^2(r^2 - 2) - 1(r^2 - 2) = 0$$

$$\Rightarrow \quad (r^2 - 2)(2r^2 - 1) = 0$$

$$\Rightarrow \quad r^2 = 2, 2r^2 = 1$$

$$\Rightarrow \quad r = \pm\sqrt{2}, r = \pm\frac{1}{\sqrt{2}}$$

On putting $r = \sqrt{2}$ in Eq. (iii), we get

$$a\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right) = 24$$

$$\Rightarrow \quad a\left(\frac{2+1}{\sqrt{2}}\right) = 24$$

$$\Rightarrow \quad 3a = 24\sqrt{2}$$

$$\Rightarrow \quad a = 8\sqrt{2}$$

$\therefore$ Series becomes $8\sqrt{2}(\sqrt{2})^3, 8\sqrt{2}(\sqrt{2}), \frac{8\sqrt{2}}{\sqrt{2}}$ and $\frac{8\sqrt{2}}{(\sqrt{2})^3}$ *i.e.*, 32, 16, 8 and 4.

If we take $r = \frac{1}{\sqrt{2}}$, we get the series 4, 8, 16 and 32.