CG PET

Engineering Entrance Exam

Solved Paper 2013

Physics

1. A given mass is suspended from a spring and time period for vertical oscillations is T_1. The spring is now cut into two equal halves and the same mass is suspended from one piece of spring. Time period of vertical oscillations is now T_2. The ratio of T_2/T_1 is
   (a) $1/2$ (b) $1/\sqrt{2}$
   (c) $\sqrt{2}$ (d) 2

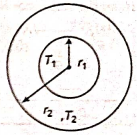
2. A bar pendulum hanged on its centre of gravity does not oscillate because
   (a) its time period is zero at that point
   (b) its time period is infinite at that point
   (c) its mass is zero at that point
   (d) its moment of inertia is zero at that point

3. In Young's double slit experiment, interference fringes are produced due to two coherent sources $2d$ metre apart. Interference pattern is observed at distance D metre apart from the sources. If λ in metre is the wavelength of light, then the number of fringes appearing per metre on screen is
   (a) $\frac{\lambda D}{d}$ (b) $\frac{\lambda D}{2d}$
   (c) $\frac{2d}{\lambda D}$ (d) $\frac{2D}{\lambda d}$

   The wavelength of H_β line of Balmer series for Hydrogen atom is
   (a) 4.9×10^7 m (b) 3.6×10^{-7} m
   (c) 4.9×10^{-7} m (d) 2.6×10^8 m

5. A prism made of glass of refractive index $\mu = \sqrt{2}$ has angle of prism 30°. Its one refracting surface is silvered. Monochromatic light beam will retrace its path when angle of incidence of this light beam on the other surface
   (a) 0° (b) 30° (c) 45° (d) 60°

6. The figure shows a system of two concentric spheres of radii r_1 and r_2 kept at temperature T_1 and T_2 respectively. The radial flow of heat in a substance between the two concentric spheres is proportional to



   (a) $(r_2 - r_1)$ (b) $\frac{r_1 r_2}{(r_2 - r_1)}$
   (c) $\ln\left(\frac{r_2}{r_1}\right)$ (d) $\frac{(r_2 - r_1)}{(r_1 r_2)}$

7. When a healthy eye is seeing an object at infinity, at that moment, values of focal length F and radius of curvature R of eye lens are
   (a) F maximum, R minimum
   (b) F minimum, R maximum
   (c) F and R both minimum
   (d) F and R both maximum

8. Two satellites X and Y are moving round the earth in the same orbit. Mass of X is twice the mass of Y, then
   (a) kinetic energies of X and Y are equal
   (b) speeds of X and Y are equal
   (c) potential energies of X and Y are equal
   (d) None of the above

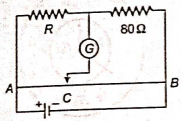
9. A black body at 1227°C emits radiation with maximum intensity at a wavelength of 5000 Å. The temperature of the body is increased by 1000°C. The maximum intensity will be observe at
   (a) 4000 Å (b) 5000 Å
   (c) 6000 Å (d) 3000 Å

10. Considering rotational motion of earth. The acceleration due to gravity at the equator of earth is given by, (where ω = angular velocity of earth, R = radius of earth, g = acceleration due to gravity)
    (a) $g' = g - \omega R$ (b) $g' = g - \omega R^2$
    (c) $g' = g - \omega^2 R$ (d) $g' = g + \omega^2 R$

11. The self-inductance of each of the two pure inductances is L. They are joined together in parallel but are isolated from each other. Total inductance will be
(a) $2L$ (b) L
(c) $L/2$ (d) $L/4$

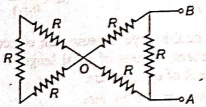
12. An electric dipole of moment **p** is lying along uniform electric field **E**. The work done in rotating the dipole by 90° is
(a) $\sqrt{2}\,\mathbf{pE}$ (b) $\frac{\mathbf{pE}}{2}$
(c) $2\,\mathbf{pE}$ (d) $\mathbf{pE}$

13. AB is a wire of uniform resistance. The galvanometer G shows no current when the length $AC = 20$ cm and $CB = 80$ cm. Then, the resistance R is equal to



(a) 12Ω (b) 16Ω
(c) 20Ω (d) 40Ω

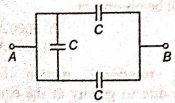
14. The equivalent resistance between A and B is



(a) $\frac{2R}{5}$ (b) $\frac{2}{7}R$
(c) $\frac{2}{3}R$ (d) $2R$

15. Entropy remains constant in
(a) isothermal process
(b) adiabatic process
(c) cyclic process
(d) isobaric process

16. The equivalent capacitance of the combination shown in the figure between A and B is



(a) $\frac{2C}{3}$ (b) $\frac{3C}{2}$
(c) $2C$ (d) $\frac{C}{2}$

17. When white light from vacuum enters glass, then in glass
(a) all the seven colours rays travel with the same speed
(b) speed of violet colour ray is more than speed of red colour ray
(c) speed of violet colour ray is less than speed of red colour ray
(d) speed of yellow colour ray is minimum and speeds of violet colour ray and red colour ray are maximum and equal

18. A long straight wire carries a current 15 A. Calculate the magnetising field H at a point at distance 0.105 m from the axis of wire
(a) 0.227 A/m (b) 2.27 A/m
(c) 22.7 A/m (d) 22.7 A-m

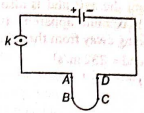
19. Consider the following two statements.
(i) Linear momentum of a body is independent of frame of reference.
(ii) Kinetic energy of a body is independent of frame of reference.
Choose the correct option.
(a) Both (i) and (ii) are false
(b) (i) is true but (ii) is false
(c) (i) is false but (ii) is true
(d) both (i) and (ii) are true

20. The correct formula used to decide the focal length of a lens is (F is focal length, μ is refractive index of the material of lens, R_1 and R_2 are radii of curvature of curved surfaces; u and v are respectively object distance and image distance
(a) $\frac{1}{F} = \frac{1}{v} - \frac{1}{u}$ (b) $\frac{1}{F} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
(c) $\frac{1}{F} = \frac{1}{v} + \frac{1}{u}$ (d) $\frac{1}{F} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

21. In photoelectric experiment, the work function of the metal is 3.5 eV. The emitted electrons can be stopped by applying a potential of –1.2 V.
Choose the correct option.
(a) The energy of incident photons is 4.7 eV
(b) The energy of incident photons is 2.3 eV
(c) If higher frequency photons are used, the photoelectric current will increase
(d) When the energy of photons is 3.5 eV, the photoelectric current value will be maximum

22. Atomic hydrogen is excited to the nth energy level. The maximum number of spectral lines which it can emit while returning to the ground state is
(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$
(c) $n(n-1)$ (d) $n(n+1)$

23. If in the below circuit, key (k) is pressed, what will be the effect on the hanging portions AB and CD of the wire?



(a) Both will attract each other
(b) Both will repel each other
(c) They will break
(d) None of the above

24. The electrical conductivity of semiconductors
(a) does not depend upon temperature
(b) increases with rise in temperature
(c) decreases with rise in temperature
(d) first decreases and then increases with rise in temperature

25. An electron enters in a magnetic field of 1.0 **k** N/A-m with a speed of $(2\mathbf{i} - 3\mathbf{j})\ \mathrm{m}^{-1}$. Calculate the Lorentz force acting on the electron
(a) $-1.6(3\mathbf{i} - 2\mathbf{j}) \times 10^{19}$ N (b) $1.6(3\mathbf{i} + 2\mathbf{j}) \times 10^{19}$N
(c) $-1.6(3\mathbf{i} + 2\mathbf{j}) \times 10^{19}$ N (d) $1.6(3\mathbf{i} - 2\mathbf{j}) \times 10^{19}$ N

26. Which of the following quantities do not change when a resistor connected to a battery is heated due to the current?
(a) Drift speed
(b) Resistivity
(c) Resistance
(d) Number of free electrons

27. For an oscillation magnetometer, the time period of suspended bar magnet can be reduced by
(a) moving it towards South pole
(b) moving it towards North pole
(c) moving it towards equator
(d) moving it towards poles

28. The electric potential at a point in free space due to charge Q coulomb is $Q \times 10^{11}$ V. The electric field at that point is
(a) $4\pi\varepsilon_0 Q \times 10^{20}$ V/m (b) $12\pi\varepsilon_0 Q \times 10^{22}$ V/m
(c) $4\pi\varepsilon_0 Q \times 10^{22}$ V/m (d) $12\pi\varepsilon_0 Q \times 10^{20}$ V/m

29. Theoretical value of Poisson's ratio is
(a) between $-\frac{1}{2}$ and $+\frac{1}{2}$
(b) between -1 and $+\frac{1}{2}$
(c) between $-\frac{1}{2}$ and $+1$
(d) between -1 and $+1$

30. Initial pressure and volume of gas are p and V respectively. First it's volume is expanded to 4 V isothermally and then again it's volume makes to be V adiabatically. Then it's final pressure is($\gamma = 1.5$)
(a) 8 p (b) 4 p
(c) p (d) 2 p

31. The penetrating power of X-rays can be increased by
(a) increasing the current in the heating filament
(b) decreasing the current in the heating filament
(c) increasing the potential difference between the cathode and anode
(d) decreasing the potential difference between the cathode and anode

32. Electric potential due to a dipole at a point R away from dipole is inversely proportional to
(a) R^2 (b) R^3
(c) $\frac{1}{R^2}$ (d) $\frac{1}{R^3}$

33. Consider a cylindrical capacitor let the length of the cylinders be L The radii of inner and outer cylinders be R_1 and R_2 and a charge $+Q$ is placed on inner cylinder and $-Q$ on the outer cylinder, then the capacitance is given by C
(a) $\frac{Q}{2\pi\varepsilon_0 l}\log\frac{R_2}{R_1}$ (b) $\frac{2\pi\varepsilon_0 l}{\log(R_2/R_1)}$
(c) $\frac{\log(R_2/R_1)}{2\pi\varepsilon_0 l}$ (d) $\frac{2\pi\varepsilon_0}{\log(R_1/R_2)}$

34. Unit of surface tension in MKS system is
(a) N-m (b) N-m^{-1}
(c) N-m^{2} (d) N-m^{-2}

35. I_A and I_B are the moment of inertia of two bodies A and B. They have same geometrical shape. If the first one A is made of gold and the second one B is made of steel, then
(a) $I_A = I_B$ (b) $I_A > I_B$
(c) $I_A < I_B$ (d) None of these

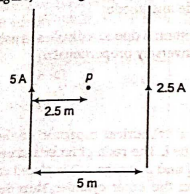
36. The pressure of an ideal gas is written as $p = \frac{2E}{3V}$, here E refers to
(a) translation kinetic energy
(b) rotational kinetic energy
(c) vibrational kinetic energy
(d) total kinetic energy

37. To increases the range of ammeter by n times, value of shunt resistance will be[$G \rightarrow$ Resistance of ammeter]
(a) $S = \frac{G}{n-1}$
(b) $S = G \times (n-1)$
(c) $S = (n-1)/G$
(d) None of these

38. Reverse bias applied to a junction diode
(a) lowers the potential barrier
(b) raises the potential barrier
(c) increases the majority carrier current
(d) increases the minority carrier current

39. In adiabatic expansion of gas, the quantity which remains constant is
(a) amount of heat
(b) temperature
(c) both the amount of heat and temperature
(d) pressure and temperature of gas

40. For the figure, the magnetic field at a point p will be



(a) $\frac{\mu_0}{4}\pi \odot$ (b) $\frac{\mu_0}{\pi} \otimes$
(c) $\frac{\mu_0}{2\pi} \otimes$ (d) $\frac{\mu_0}{2}\pi \odot$

41. Magnifying power of an astronomical telescope will be maximum when the final image formed by it is
(a) at infinity
(b) atleast distance of distinct vision
(c) at anywhere
(d) at optical centre of objective lens

42. A block of mass M is pulled by a force F in the direction at an angle θ from the horizontal surface. Friction coefficient between block and surface is μ. The value of force F is
(a) μMg (b) $\frac{\mu Mg}{1+\mu \sin\theta}$
(c) $\frac{\mu Mg}{\cos\theta + \mu \sin\theta}$ (d) None of the above

43. Faraday constant F is given by
(a) $\frac{\text{Atomic weight}}{\text{Valency}}$
(b) $\frac{\text{Chemical equivalent}}{\text{Electro chemical equivalent}}$
(c) Avogadro number × charge of one electron
(d) $\frac{\text{Avogadro number}}{\text{Charge of one electron}}$

44. A table is revolving about a vertical axis passing through its centre at 5 revolutions per sec. A sound source of frequency 1000 Hz is fixed on the table at 70 cm away from the axis and is also revolving with the table. The maximum apparent frequency heard by listener standing away from the table will be (speed of sound = 332 m/s)
(a) 1000 Hz
(b) 1071 Hz
(c) 938 Hz
(d) 1066 Hz

45. A thin prism P_1 with angle 6° and made from glass of refractive index 1.54 is combined with another thin prism P_2 of glass of refractive index 1.72 to produce dispersion without deviation. The value of angle of prism of P_2 will be
(a) 5°-24′ (b) 4°-30′
(c) 6° (d) 8°

46. A resistor of resistance R is connected to an ideal battery. If the value of R is increased the power dissipated in the resistor will
(a) increase (b) decrease
(c) no change (d) None of these

47. The quantity, which remain unchanged in a transformer, is
(a) voltage (b) current
(c) frequency (d) None of these

48. In a lift moving upward weight of a man is 708 N. While in a lift moving downwards (with uniform acceleration weight of same man is 468 N. Normal weight of man is
(a) 608 N (b) 478 N
(c) 588 N (d) 508 N

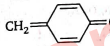
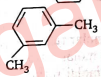
49. During an adiabatic process, the pressure of a gas is found proportional to the cube of its absolute temperature. The ratio of C_p/C_V for the gas is
(a) $\frac{4}{3}$ (b) 2
(c) $\frac{5}{3}$ (d) $\frac{3}{2}$

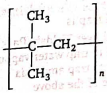
50. A set of 56 tuning forks are so arranged in series that each fork gives 4 beats per second with the previous one. The frequency of last fork is 3 times that of the first. Frequency of first fork is
(a) 56 Hz
(b) 60 Hz
(c) 120 Hz
(d) 110 Hz

Chemistry

1. Which one of the following has minimum gold number?
(a) Starch (b) Sodium oleate
(c) Gelatin (d) Gum arabic

2. The reaction,
$3ClO^-(aq) \longrightarrow ClO_3^-(aq) + 2Cl^-(aq)$ is an example of
(a) oxidation reaction
(b) reduction reaction
(c) disproportionation reaction
(d) decomposition reaction

3. Which compound is aromatic?
(a) 
(b) 
(c) 
(d) 

4. Monomer of polymer 
(a) 2-methylpropene (b) styrene
(c) propene (d) ethene

5. Which gas behaves abnormally when liquified?
(a) Xenon (b) Krypton
(c) Helium (d) Argon

6. A radioactive element has half-life 150 yr. A sealed tube containing 1.0 g of sample will contain after 300 yr will be
(a) 1.0 g (b) 0.5 g
(c) 0.25 g (d) 0.125 g

7. The chemical composition of slag formed during the smelting process in the extraction of copper is
(a) $FeSiO_3$ (b) $CuFeS_2$
(c) $Cu_2S + FeO$ (d) $Cu_2O + FeS$

8. The substance not likely to contain $CaCO_3$ is
(a) marble statue (b) calcined gypsum
(c) sea shells (d) dolomite

9. 5 millimoles of caustic potash and 5 millimoles of oxalic acid are mixed and dissolved in 100 mL water. The solution will be
(a) basic (b) acidic
(c) neutral (d) can't say

10. CO_2 acts as electrophile in which reaction?
(a) Williamson's reaction
(b) Kolbe reaction
(c) Perkin's reaction
(d) Reimer- Tiemann reaction

11. Which of the following will have the shape of a trigonal bipyramid?
(a) PF_3Cl_2 (b) IF_5 (c) BrF_5 (d) SbF_5^{2-}

12. The example of σ-complex is
(a) $Fe(\eta^5—C_5H_5)_2$ (b) $[Cr(CO)_6]$
(c) $Al_2(CH_3)_6$ (d) Ziess salt

13. How many moles of O_2 can be obtained by electrolysis of 90 g H_2O?
(a) 5.0 mol (b) 0.5 mol
(c) 2.5 mol (d) 0.25 mol

14. Which one of the following undergoes aldol condensation?
(a) Acetaldehyde (b) Propanaldehyde
(c) Acetone (d) All of these

15. Schiff's nitrometre is filled with
(a) mercury
(b) water over mercury seal
(c) KOH solution over mercury seal
(d) toluene over mercury seal

16. Four different sets of quantum numbers for 4 electrons are given below
$e_1 = 4, 0, 0, -\frac{1}{2}$
$e_2 = 3, 1, 1, -\frac{1}{2}$
$e_3 = 3, 2, 2, +\frac{1}{2}$
$e_4 = 3, 0, 0, +\frac{1}{2}$
The order of energy of e_1, e_2, e_3, e_4 is
(a) $e_1 > e_2 > e_3 > e_4$ (b) $e_4 > e_3 > e_2 > e_1$
(c) $e_3 > e_1 > e_2 > e_4$ (d) $e_2 > e_3 > e_4 > e_1$

17. Silver ornaments turn black in the atmosphere. It is due to the formation of
(a) Ag_2O, Ag_2S (b) $AgNO_3, Ag_2S$
(c) $Ag(OH), Ag_2CO_3$ (d) Ag, Ag_2O

18. How much Coulomb needed to convert 1 mole of MnO_4 into Mn^{2+}?
(a) 482500 C (b) 193000 C
(c) 96500 C (d) 36500 C

19. Except one, the other three are isomers, find odd man out

ethanol	oxiran	oxitane	vinyl alcohol
(1)	(2)	(3)	(4)

(a) vinyl alcohol (b) ethanol
(c) oxiran (d) oxitane

20. Which of the following nuclei is most unstable?
(a) $^{40}_{20}Ca$ (b) $^{55}_{35}Mn$ (c) $^{119}_{50}Sn$ (d) $^{30}_{13}Al$

21. The geometrical and optical isomers of complex $[Pt(NH_3)(Br)(Cl)(Py)]$ are respectively
(a) 2, 2 (b) 0, 3
(c) 2, 1 (d) 3, 0

22. One amine is more basic than ammonia and the other is less basic than ammonia. The two amines are respectively
(a) N–methyl ethanamine and N, N-dimethyl ethanamine
(b) aniline and N-methyl aniline
(c) N-methyl aniline and aniline
(d) N, N-dimethyl aniline and benzenamine

23. Aluminium is extracted by the electrolysis of
(a) alumina
(b) bauxite
(c) molten cryolite
(d) alumina mixed with molten cryolite

24. For the reaction,
$$2NO + Br_2 \longrightarrow 2NOBr,$$
the following mechanism has been given
$$NO + Br \xrightarrow{fast} NOBr_2$$
$$NOBr_2 + NO \xrightarrow{slow} 2NOBr$$
Hence rate law is
(a) $k[NO]^2[Br_2]$ (b) $k[NO][Br_2]$
(c) $k[NOBr_2][NO]$ (d) $k[NO][Br_2]^2$

25. Electron affinity is positive when
(a) O^- is formed from O
(b) O^{2-} is formed from O^-
(c) O^+ is formed from O
(d) electron affinity is always a negative value

26. When solid melts, there will be
(a) a decrease in enthalphy
(b) a decrease in free energy
(c) a decrease in entropy
(d) all the above factors remain constant

27. Which of the following substance is not related with HVZ reaction?
(a) α-bromo acetic acid (b) Zn/Hg
(c) Cl_2 (d) Red P

28. The age of most ancient geological formation is estimated by
(a) carbon dating (b) potassium-argon dating
(c) radium-radon dating (d) uranium-lead dating

29. One of the isomer of $C_4H_{11}N$ is optically active. It must be a
(a) primary amine
(b) secondary amine
(c) tertiary amine
(d) all isomers are optically inactive

30. The electronic configuration of an element C is $1s^2, 2s^2, 2p^6$. The formula of substance containing only C will be
(a) C_8 (b) C_4 (c) C_2 (d) C

31. Which solution is a buffer?
(a) Acetic acid + NaOH (equimolar ratio)
(b) Acetic acid + NaOH (1 : 2 molar ratio)
(c) Acetic acid + NaOH (2 : 1 molar ratio)
(d) HCl + NaOH (equimolar ratio)

32. Which of the following compound show acidic nature?
(a) but-1-yne (b) but-2-yne
(c) but-1-ene (d) buta-1, 3-diene

33. Kjeldahl trap is
(a) fitted over Kjeldahl flask
(b) used to trap water vapours
(c) used to trap ammonia
(d) None of the above

34. The bond present in N_2O_5 are
(a) only ionic (b) covalent and coordinate
(c) only covalent (d) covalent and ionic

35. The hydration energy of Mg^{2+} ions is higher than that of
(a) Al^{3+} (b) Be^{2+}
(c) Na^+ (d) None of these

36. Bakelite is polymer of phenol and
(a) HCOOH (b) $HCOOCH_3$
(c) HCHO (d) CH_3COOH

37. The solubility of $Al(OH)_3$ is 'S' mol L^{-1}, The solubility product will be
(a) S^2 (b) S^3 (c) $27S^4$ (d) $27S^3$

38. Give the increasing order of stability in the following complexes ions
$[AlF_6]^{3-}$ $[Cd(CN)_4]^{2-}$ $[Ag(CN)_2]^-$ $[Zn(CN)_4]^{2-}$
A B C D
(a) $D < C < B < A$ (b) $A < D < B < C$
(c) $A < B < C < D$ (d) None of these

39. Which one of the following transition metal ion is coloured?
(a) Cu^+ (b) Zn^{2+}
(c) Sc^{3+} (d) V^{4+}

40. Which of the following is involved in formation of heme?
(a) Lysine (b) Glycine
(c) Tryosine (d) Arginine

41. Sodium atom crystallizes in bcc lattice with cell edge a = 4.29 Å, the radius of sodium atom is
(a) 18.6 Å (b) 1.86 Å
(c) 0.186 Å (d) 37.2 Å

42. See the following redox reaction
$A^{2+} + 2e^- \longrightarrow A; E^0 = +0.34$ V
$A^+ + e^- \longrightarrow A; E^0 = +0.52$ V
Which ion is expected to be stable?
(a) A^{2+}
(b) A^+
(c) Both can form stable complexes
(d) None can form stable complexes

43. When 1 M H_2SO_4 is completely neutralized by NaOH, the heat liberated is 114.64 kJ. What is the enthalpy of neutralization?
(a) + 114.64 kJ (b) –114.64 kJ
(c) –57.32 kJ (d) + 57.32 kJ

44. The first ionisation potential (in eV) of N and O atoms are
(a) 14.6, 13.6
(b) 13.6, 14.6
(c) 13.6, 13.6
(d) 14.6, 14.6

45. One of the isomer of the 5th member of alkyne series is optically active. It is
(a) 4-methyl pent-2-yne (b) 3-methyl pent-1-yne
(c) 4-methyl pent-1-yne (d) 3, 3-dimethyl but-1-yne

46. The aqueous ferrous ion is green, ferric ion is pale yellow. The aqueous chromic ion is green, hence the colour of chromate ion must be
(a) orange (b) colourless
(c) yellow (d) red

47. $\frac{K_p}{K_c}$ for the reaction
$CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$ is
(a) RT (b) $\frac{1}{\sqrt{RT}}$
(c) $\sqrt{RT}$ (d) $\frac{1}{RT}$

48. The human body does not produce
(a) enzyme (b) DNA
(c) vitamins (d) hormones

49. The dimensions of a unit cell of a crystal are $a = 0.397$, $b = 0.387$, $c = 0.504$ and $\alpha = \beta = 90°$, $\gamma = 120°$ the crystal system is
(a) cubic (b) hexagonal
(c) orthorhombic (d) rhombohedral

50. A particle 'A' moving with a certain velocity has the de-Broglie wavelength 1 Å. For a particle 'B' with mass 25% of 'A' and velocity 75% of 'A'. The de-Broglie wave length of 'B' will be
(a) 3 Å (b) 5.33 Å
(c) 6.88 Å (d) 0.68 Å

Mathematics

1. The area of the parallelogram having the diagonals $3i + j - 2k$ and $i - 3j + 4k$ is
(a) 5 sq units (b) $10\sqrt{3}$ sq units
(c) $5\sqrt{3}$ sq units (d) 10 sq units

2. The value of $(i + j)\cdot[(j + k)\times(k + i)]$ is
(a) 0 (b) 1 (c) –1 (d) 2

3. The solution of the differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is
(a) $x\sin\left(\frac{x}{y}\right) + C = 0$
(b) $x\sin y + C = 0$
(c) $x\sin\left(\frac{y}{x}\right) = C$
(d) None of these

4. The angle of elevation of the Sun, when the shadow of the pole is $\sqrt{3}$ times the height of the pole, is
(a) 60° (b) 45°
(c) 15° (d) 30°

5. The curves $x = y^2$ and $xy = k$ cut at right angles, if k^2 is equal to
(a) 1 (b) 0
(c) $\frac{1}{8}$ (d) 8

6. If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then $\sin(\theta/2)$ is equal to
(a) $\frac{|\hat{a} + \hat{b}|}{2}$ (b) $\frac{\hat{a} - \hat{b}}{2}$
(c) $\frac{|\hat{a} - \hat{b}|}{2}$ (d) $|\hat{a} - \hat{b}|$

7. If α and β are roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(a) $\frac{a^2}{2}(\alpha-\beta)^2$ (b) $-\frac{a^2}{2}(\alpha-\beta)^2$
(c) 0 (d) 1

8. The particular solution of $\cos\left(\frac{dy}{dx}\right)=a$ (where, $a\in R$), ($y=2$ when $x=0$), is

(a) $\cos\left(\frac{y-2}{x}\right)=a$ (b) $\sin\left(\frac{y-2}{x}\right)=a$
(c) $\cos^{-1}x=y+a$ (d) $y=a\cos^{-1}x$

9. The square root of $2i$ is

(a) $1+i$ (b) $1-i$
(c) $\sqrt{2}i$ (d) $-\sqrt{2}$

10. The locus of z given by $\left|\frac{z-1}{z+1}\right|=1$ is

(a) a parabola (b) an ellipse
(c) a circle (d) a straight line

11. If $\sin\theta=\frac{\sqrt{3}}{2}$, then the general value of θ is

(a) $n\pi+(-1)^n\frac{\pi}{3}$ (b) $2n\pi\pm\frac{\pi}{6}$
(c) $2n\pi\pm\frac{\pi}{3}$ (d) $n\pi+(-1)^n\frac{\pi}{6}$

12. The root of the equation $2x-\log_{10}x=7$ is between

(a) 3 and 3.5 (b) 2 and 3
(c) 3.5 and 4 (d) None of these

13. The coefficient of correlation between x and y is 0.8, whereas the regression coefficient of y on x is 0.2, then the regression coefficient of x on y will be

(a) −3.2 (b) 3.2
(c) 4 (d) 0.16

14. If $\sin(x+y)=\log(x+y)$, then $\frac{dy}{dx}$ is equal to

(a) −1 (b) 1
(c) 2 (d) −2

15. A conic section represents a circle, if its eccentricity e is

(a) $e<0$ (b) $e>0$
(c) $e=0$ (d) None of these

16. $\frac{d}{dx}\cot^{-1}x$ is equal to

(a) $\frac{1}{1+x^2}$ (b) $\frac{-1}{1+x^2}$
(c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{-1}{\sqrt{1+x^2}}$

17. The value of $\tan 9^\circ-\tan 27^\circ-\tan 63^\circ+\tan 81^\circ$ is

(a) 3 (b) 2
(c) 8 (d) 4

18. The number of terms in the series $105+103+101+\ldots+49+47$ is

(a) 28 (b) 30
(c) 25 (d) 22

19. If the variance of two variables x and y are respectively 9 and 16 and covariance is 8, then the coefficient of correlation between x and y will be

(a) $\frac{2}{3}$ (b) $\frac{8}{3\sqrt{2}}$ (c) $\frac{9}{8\sqrt{2}}$ (d) $\frac{2}{9}$

20. If $A=\begin{bmatrix}1&0&1\\0&1&1\\1&0&0\end{bmatrix}$, then A is a

(a) singular (b) non-singular
(c) symmetric (d) unit matrix

21. The ratio in which yz-plane divide the line joining the points $A(3,1,-5)$ and $B(1,4,-6)$ is

(a) −3 : 1 (b) 3 : 1
(c) −1 : 3 (d) 1 : 3

22. If ${}^nP_5=20\,{}^nP_3$, then the value of n is

(a) 7 (b) 5
(c) 8 (d) 9

23. The order and degree of the differential equation $\frac{d^2y}{dx^2}=\left\{y+\left(\frac{dy}{dx}\right)^2\right\}^{1/4}$ are given by

(a) 4 and 2 (b) 1 and 2
(c) 1 and 4 (d) 2 and 4

24. The differential equation of the family of circles touching the y-axis at the origin is

(a) $xy'-2y=0$
(b) $y''-4y'+4y=0$
(c) $2xyy'+x^2=y^2$
(d) $2yy'+y^2=x^2$

25. $\int\frac{dx}{x^4-1}$ is equal to

(a) $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$
(b) $\log\left|\frac{x-1}{x+1}\right|+C$
(c) $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{2}\tan^{-1}x+C$
(d) $\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$

26. If the points $(k,2-2k)$, $(-k+1,2k)$ and $(-4-k,6-2k)$ are collinear, then k is equal to

(a) −1 (b) $\frac{1}{3}$
(c) 1 (d) $-\frac{1}{2}$

27. The equation of a straight line parallel to the x-axis is given by

(a) $\frac{x-a}{1}=\frac{y-b}{1}=\frac{z-c}{1}$ (b) $\frac{x-a}{0}=\frac{y-b}{0}=\frac{z-c}{1}$
(c) $\frac{x-a}{0}=\frac{y-b}{1}=\frac{z-c}{1}$ (d) $\frac{x-a}{1}=\frac{y-b}{0}=\frac{z-c}{0}$

28. If $\log_{10^4}x=y$, then $\log_{10^8}x^4$ is equal to

(a) $\frac{2}{3}y$ (b) $3y$ (c) $4y$ (d) $2y$

29. The shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ is

(a) $\frac{1}{2}\sqrt{29}$ units (b) $2\sqrt{29}$ units
(c) $\sqrt{29}$ units (d) $\frac{1}{4}\sqrt{29}$ units

30. The largest term in the expansion of $(3+2x)^{50}$, where $x=\frac{1}{5}$, is

(a) 7th (b) 5th (c) 8th (d) 49th

31. If $x=a\cos^3t$ and $y=a\sin^3t$, then $\left(\frac{dy}{dx}\right)_{t=\pi/4}$ is equal to

(a) 1 (b) −1 (c) 0 (d) ∞

32. The anti-derivative F of f defined by $f(x)=4x^3-6x^2+2x+5$, where $F(0)=5$, is

(a) $x^4-2x^3+x^2+5x$
(b) $12x^2-12x+2$
(c) $16x^4-18x^3+4x^2+5x$
(d) $x^4-2x^3+x^2+5x+5$

33. Scanner is

(a) an input device (b) an output device
(c) a memory device (d) None of the above

34. The value of $\sqrt{12}$ upto three places of decimals using the method of Newton-Raphson, will be

(a) 3.463 (b) 3.462
(c) 3.467 (d) None of these

35. The geometric mean of roots of the equation $x^2-18x+81=0$ is

(a) 18 (b) 6 (c) 9 (d) 3

36. $\lim_{x\to 0}x(-1)^{\left[\frac{1}{x}\right]}$ is equal to

(a) 0 (b) 1
(c) −1 (d) Does not exist

37. Area of the region bounded by the parabola $y=x^2$ and the curve $y=|x|$ is

(a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$

38. The equation of circle passing through the points (0, 2), (3, 3) and having its centre on the x-axis is

(a) $x^2+y^2-14x-12=0$
(b) $3x^2+3y^2-22x-4=0$
(c) $3x^2+3y^2-14x-12=0$
(d) None of the above

39. If $\sin\theta=\frac{1}{2}$ and where, θ is an obtuse angle, then $\cot\theta$ is equal to

(a) $-\frac{1}{\sqrt{3}}$ (b) $-\sqrt{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

40. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2};z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3};\frac{z+5}{2}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

41. RAM (Random Access Memory) in a computer is

(a) main memory (b) secondary memory
(c) Both of them (d) None of the above

42. If the total cost $C(x)$ in rupees associated with the production of x units of an item is given by $C(x)=3x^3-2x^2+x+100$. Then, the marginal change in cost, when $x=5$, is

(a) 200 (b) 225
(c) 206 (d) 226

43. If two events A and B are mutually exclusive events, then $P(A/B)$ is equal to

(a) 0 (b) 1
(c) $\frac{P(A\cap B)}{P(A)}$ (d) $\frac{P(A\cap B)}{P(B)}$

44. If **a**, **b** and **c** are three non-zero, non-coplanar vectors, then the value of $\mathbf{a}\times\mathbf{a}'+\mathbf{b}\times\mathbf{b}'+\mathbf{c}\times\mathbf{c}'$ is

(a) 1 (b) 0
(c) −1 (d) None of the above

45. If $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.60$, then the value of $\int_0^4 e^x\,dx$ using Simpson's rule, will be

(a) 5.387 (b) 53.87
(c) 52.78 (d) 53.17

46. The roots of the quadratic equation $2x^2 + 3x + 1 = 0$ are

(a) rational (b) irrational
(c) imaginary (d) None of these

47. A and B are two independent events. Probability of happening of both A and B is 1/6 and probability of happening of neither of them is 1/3, then the probability of events A and B are respectively

(a) $\frac{1}{2}$ and $\frac{1}{3}$ (b) $\frac{1}{5}$ and $\frac{1}{6}$
(c) $\frac{1}{2}$ and $\frac{1}{6}$ (d) $\frac{2}{3}$ and $\frac{1}{4}$

48. The equation of a circle passing through origin and radius is a, is

(a) $(x-a)^2 + (y-a)^2 = a^2$
(b) $x^2 + y^2 = a^2$
(c) $(x-a)^2 + y^2 = a^2$
(d) None of the above

49. Compiler in a computer is

(a) an application software
(b) a system software
(c) a package
(d) a tool

50. $\int_{-1}^{2} |x^3 - x|\,dx$ is equal to

(a) 11 (b) 4
(c) $\frac{11}{4}$ (d) $\frac{4}{11}$

Answers

Physics

1. (b)	2. (b)	3. (c)	4. (c)	5. (c)	6. (b)	7. (d)	8. (b)	9. (d)	10. (c)
11. (c)	12. (d)	13. (c)	14. (c)	15. (b)	16. (b)	17. (c)	18. (c)	19. (a)	20. (a)
21. (b)	22. (a)	23. (b)	24. (b)	25. (b)	26. (d)	27. (c)	28. (c)	29. (b)	30. (b)
31. (c)	32. (a)	33. (b)	34. (b)	35. (b)	36. (a)	37. (a)	38. (b)	39. (a)	40. (c)
41. (b)	42. (c)	43. (c)	44. (a)	45. (b)	46. (b)	47. (c)	48. (c)	49. (d)	50. (d)

Chemistry

1. (c)	2. (c)	3. (c)	4. (a)	5. (c)	6. (c)	7. (a)	8. (b)	9. (b)	10. (b)
11. (a)	12. (c)	13. (c)	14. (d)	15. (c)	16. (c)	17. (a)	18. (a)	19. (b)	20. (d)
21. (d)	22. (a)	23. (d)	24. (a)	25. (b)	26. (b)	27. (b)	28. (d)	29. (a)	30. (d)
31. (c)	32. (a)	33. (b)	34. (b)	35. (c)	36. (c)	37. (c)	38. (d)	39. (d)	40. (b)
41. (b)	42. (a)	43. (c)	44. (a)	45. (b)	46. (c)	47. (b)	48. (c)	49. (b)	50. (b)

Mathematics

1. (c)	2. (d)	3. (c)	4. (d)	5. (c)	6. (c)	7. (a)	8. (a)	9. (a)	10. (d)
11. (a)	12. (c)	13. (b)	14. (a)	15. (c)	16. (b)	17. (d)	18. (b)	19. (a)	20. (b)
21. (a)	22. (c)	23. (d)	24. (c)	25. (a)	26. (a)	27. (d)	28. (d)	29. (b)	30. (a)
31. (b)	32. (d)	33. (a)	34. (a)	35. (c)	36. (a)	37. (b)	38. (c)	39. (b)	40. (c)
41. (a)	42. (c)	43. (a)	44. (b)	45. (b)	46. (a)	47. (a)	48. (c)	49. (b)	50. (c)

Hints & Solutions

Physics

1. If a spring of spring constant k is cut into two equal parts, then spring constant of each part is $2k$.

Given, in Ist case $T_1 = 2\pi\sqrt{\frac{m}{k}}$

In second case, $T_2 = 2\pi\sqrt{\frac{m}{2k}}$

$$\therefore \quad \frac{T_2}{T_1} = \frac{2\pi\sqrt{m/2k}}{2\pi\sqrt{m/k}} = \frac{1}{\sqrt{2}}$$

2. It does not oscillate because its time period becomes infinite at that point, as $T = 2\pi\sqrt{\frac{I}{Mgl}}$, where l is the distance between the centres of suspension and CG of the pendulum.

3. In Young's double slit experiment, fringe width is $\frac{\lambda D}{d}$. Let n be the number of fringes appearing per metre.

Given, $d = 2d$, $D = D$

$$\therefore \quad 1 = \frac{n\lambda D}{2d}$$

$$\Rightarrow \quad n = \frac{2d}{\lambda D}$$

4. By the relation, $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

where, $R = 1.097 \times 10^7\ m^{-1}$ (Rydberg constant)

For H_β line of Balmer series, we have, $n = 4$

$$\therefore \quad \frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 486.1\ nm$$

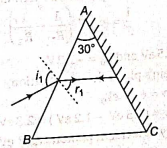
$$= 4.86 \times 10^{-7}\ m$$

$$\approx 4.9 \times 10^{-7}\ m$$

5. Given, $A = 30°$, $\mu = 1.414$, $i_1 = ?$

The refracted ray will retrace its path, when

$$i_2 = 0°,\ r_2 = 0°$$



So, $r_1 + r_2 = A$

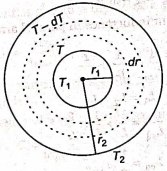
$\Rightarrow \quad r_1 + 0 = 30°$ or $r_1 = 30°$

From, $\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sin i_1 = \mu \sin r_1$

$$= 1.414 \sin 30° = 0.707$$

$$\therefore \quad i_1 = 45°$$

6. Consider a shell of thickness (dr) and of radii (r) and the temperature of inner and outer surfaces of this shell be $T, (T - dT)$.



$$\frac{dQ}{dt} = \text{rate of flow of heat through it}$$

$$= \frac{kA[(T - dT) - T]}{dr} = \frac{-KAdT}{dr}$$

$$= -4\pi kr^2\frac{dT}{dT} \qquad [\because A = 4\pi r^2]$$

To measure the radial rate of heat flow, integration technique is used, since the area of the surface through which heat will flow is not constant.

Then,

$$\left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2}\frac{1}{r^2}\,dr = -4\pi K\int_{T_1}^{T_2}dT$$

$$\frac{dQ}{dt}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = -4\pi K\,(T_2 - T_1)$$

or $$\frac{dQ}{dt} = \frac{-4\pi k_1 r_2(T_2 - T_1)}{(r_2 - r_1)}$$

$$\Rightarrow \quad \frac{dQ}{dt} \propto \frac{(r_1 r_2)}{(r_2 - r_1)}$$

7. For a healthy eye seeing an object at infinity, the focal length (F) and radius of curvature (R) both will be maximum.

8. We know that, for a satellite moving around the earth, the orbital speed is independent of the mass of the satellite. Hence, for satellite X and Y moving in the same orbit around the earth, speeds are equal.

9. By Wien's displacement law

$$\lambda T = \text{constant}$$

Here, $\lambda_1 = 5000\ Å$, $T_1 = 1227°C$

$$= (1227 + 273)\,K = 1500\ K$$

$$\lambda_2 = ?,\ T_2 = (1227 + 1000)°C$$

$$= (2227 + 273)\,K = 2500\ K$$

$$\therefore \quad \lambda_1 T_1 = \lambda_2 T_2$$

or $$\lambda_2 = \frac{\lambda_1 T_1}{T_2} = \frac{5000 \times 1500}{2500} = 3000\ Å$$

10. Due to rotation of earth, the effective value of g is

$$g' = g - \omega^2 R\cos^2\lambda$$

where, ω = angular velocity of earth's rotation and λ = latitude of given place

Here, $\lambda = 0$

$$g' = g - \omega^2 R\cos^2 0$$

$$\Rightarrow \quad g' = g - \omega^2 R$$

11. The equivalent inductance in parallel arrangement is given as

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Here, $L_1 = L_2 = L$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L} + \frac{1}{L} = \frac{2}{L}$$

So, $L_{eq} = \frac{1}{2}$

12. Work done to rotate a dipole placed in a uniform electric field is given as, $W = \mathbf{PE}(\cos\theta_1 - \cos\theta_2)$

Here, $p = P, \theta_1 = 0°$ and $\theta_2 = 90°$

$$\therefore \quad W = \mathbf{PE}(\cos 0° - \cos 90°) = \mathbf{PE}$$

13. The given arrangement represents a metre bridge, which works on the Wheatstone bridge principle

$$\frac{P}{Q} = \frac{R}{S}$$

Here, $P = 20\text{ cm} = 0.2\text{ m}$

$Q = 80\text{ cm} = 0.8\text{ m}$

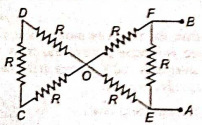
R = unknown resistance

$S = 80\,\Omega$

$$\therefore \quad \frac{0.2}{0.8} = \frac{R}{80}$$

$$\Rightarrow \quad R = \frac{0.2 \times 80}{0.8} = 20\,\Omega$$

14. In the given circuit, due to junction at O, the current will not flow in the arms OC, CD and DO. So, arms OE and OF are in series to each other and their combination is parallel with aim EF



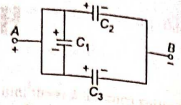
$$\therefore \quad \frac{1}{R_{eff}} = \frac{1}{2R} + \frac{1}{R'} = \frac{1+2}{2R} = \frac{3}{2}R$$

$$\Rightarrow \quad R_{eff} = \frac{2}{3}R$$

15. In an adiabatic process, p, V and T change but $\Delta Q = 0$.

Hence, entropy, $\Delta S = \frac{\Delta Q}{T} = 0$ remains constant.

16. The given circuit can be shown as



The capacitors C_1 and C_3 are in series,

Given that $C_1 = C_2 = C_3 = C$

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C'} = \frac{1}{C} + \frac{1}{C}$$

$$\Rightarrow \quad C' = \frac{C}{2}$$

Now, C' and C_2 are in parallel with each other. Hence,

$$C_{eq} = C' + C = \frac{C}{2} + C = \frac{3}{2}C$$

17. In vacuum, all electromagnetic waves travel at the same speed. So, red light and violet light travel at the same speed in vacuum. In glass, however they travel at different speeds. Since μ_R is less than μ_V, so speed of red ray is greater than the speed of violet ray in glass.

18. For a long straight wire, $B = \frac{\mu_0 I}{2\pi r}$

Here, $I = 15$ A and $r = 0.105$ m

The magnetising field (H) is given as,

$$H = \frac{B}{\mu_0} = \frac{\mu_0 I}{2\pi r \times \mu_0}$$

$$= \frac{15}{2 \times 3.14 \times 0.105} = \frac{15}{0.6594}$$

$$= 22.7\text{ A/m}$$

19. Both statements are false since linear momentum and kinetic energy of a body depends on the choice of frame of reference.

20. Remember, lens formula is used to decide the focal length of a lens which is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$.

This is different from lens makers formula

$$\frac{1}{F} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

21. Given, $\phi = 3.5$ eV and $V_0 = -1.2$ V

By the Einstein's photoelectric equation

$$h\nu = \phi_0 + eV_0$$

$$\Rightarrow \quad h\nu = (3.5\text{ eV} - 1.2\text{ eV}) = 2.3\text{ eV}$$

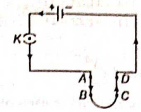
Hence, energy of incident photon is 2.3 eV.

If the higher frequency photons are used, photoelectric current remains unchanged and photoelectric current will be maximum. If the intensity of the incident light increases.

22. The possible number of emission lines between n levels is given by

$$\frac{n(n-1)}{2}$$

23. The given circuit is shown as



As the currents in the portions AB and CD are in opposite directions, so both will repel each other.

24. Electrical conductivity,

$$\sigma = \frac{1}{\rho} = e(n_e\mu_e + n_h\mu_h)$$

where, n_e = free electron density, n_h = hole density and μ_e and μ_h are their mobilities respectively. As the temperature increases, n_e and n_h increase, hence conductivity of semiconductor increases.

25. Here, $\mathbf{v} = (2\mathbf{i} - 3\mathbf{j})$ m/s and

$\mathbf{B} = 1.0\,\mathbf{k}$ N/A-m

Lorentz force, $\mathbf{F} = e(\mathbf{v} \times \mathbf{B})$

$$= (-1.6 \times 10^{-19})(2\mathbf{i} - 3\mathbf{j}) \times (1\mathbf{k})]$$

$$= (-1.6 \times 10^{-19})[2(\mathbf{i} \times \mathbf{k}) - 3(\mathbf{j} \times \mathbf{k})]$$

$$= (-1.6 \times 10^{-19})[2(-\mathbf{j}) - 3(\mathbf{i})]$$

$$= 1.6(3\mathbf{i} + 2\mathbf{j}) \times 10^{-19}\text{ N}$$

26. When a resistor connected to a battery is heated up due to current then number of free electrons do not change with rise in temperature.

27. Time period of vibration magnetometer,

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

At equator, B_H is maximum, so time period is less.

By moving it towards equator.

28. Given, $V = \frac{KQ}{r} = Q \times 10^{11}$ V

$$\Rightarrow \quad r = \frac{KQ}{Q \times 10^{11}} = \frac{K}{10^{11}}\text{ m}$$

The electric field at that point,

$$E = \frac{KQ}{r^2} = \frac{KQ}{(K/10^{11})^2}\text{ V/m}$$

$$= \frac{KQ}{K^2} \times 10^{22}\text{ V/m}$$

$$= 4\pi\epsilon_0 \times Q \times 10^{22}\text{ V/m}$$

29. We have $Y = 3K(1 - 2\sigma)$, $Y = 2n(1 + \sigma)$

For $Y = 0$, we get $(1 - 2\sigma) = 0$ and $(1 + \sigma) = 0$

$\Rightarrow$ σ lies between $\frac{1}{2}$ and -1.

30. For isothermal expansion

$$pV = p_1 \times 4V \quad \text{...(i)}$$

$$\Rightarrow \quad p_1 = \frac{p}{4}$$

For adiabatic expansion

$$pV^\gamma = p_f(V)^\gamma$$

$$\Rightarrow \quad p_f = p \quad \text{...(ii)}$$

From Eq. (i) $p_f = 4p_i$

So, the final pressure is four times of the initial pressure.

31. Penetrating power of X-rays can be increased by increasing the potential difference between the cathode and target.

32. Electric potential due to a dipole at a point R away from dipole is

$$V = \frac{p\cos\theta}{4\pi\epsilon_0 R^2}$$

So, potential varies inversely as the square of distance from the dipole.

33. The capacitance of a cylindrical capacitor is given as

$$C = \frac{2\pi\epsilon_0 l}{\log_e(R_2/R_1)}$$

or

$$C = \frac{2\pi\epsilon_0 l}{2.303\log_{10}(R_2/R_1)}$$

34. Unit of surface tension in MKS system is N/m.

35. As the geometrical shape of the two bodies is same, so both of them have same volumes.

So, the mass of body $A = V_A \cdot \rho_A$

and the mass of body $B = V_B \cdot \rho_B$

The moment of inertia of body A,

$$I_A = (V_A\rho_A)(\text{distance})^2$$

and the moment of inertia of body B,

$$I_B = (V_B\rho_B)(\text{distance})^2$$

As the volume and distance of both the bodies are same, then

$$\therefore \quad I \propto \rho$$

So, body having higher density have greater moment of inertia.

As the density of gold is greater than that of steel

So, $I_A > I_B$

36. In $P = \frac{2E}{3V}$, E refers to translational kinetic energy.

37. The range of ammeter is increased by n times by connecting a shunt in parallel having value

$$S = \frac{G}{(n-1)}$$

38. When a junction diode is reverse biased, the potential barrier increases.

39. In adiabatic expansion of gas, no heat is allowed to enter into or escape from the gas. So, amount of heat remains constant in an adiabatic expansion of gas.

40. Magnetic field due to wire carrying current 5 A is

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 \times 5}{2\pi \times 2.5} = \frac{\mu_0}{\pi}\ \otimes$$

Magnetic field due to wire carrying current 2.5 A is

$$B_2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 \times 2.5}{2\pi \times 2.5} = \frac{\mu_0}{2\pi}.$$

Net magnetic field at P, $B = B_1 - B_2$

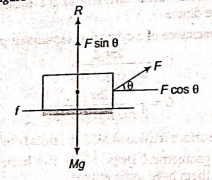
$$= B_1 - B_2 = \left(\frac{\mu_0}{\pi} - \frac{\mu_0}{2\pi}\right)$$

$$= \frac{\mu_0}{2\pi} \otimes$$

41. When final image is atleast distance of distinct vision, magnifying power is maximum, *i.e.*,

$$MP = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{d}\right)$$

42. From figure



$$f = F\cos\theta$$

$$\mu R = F\cos\theta$$

$\therefore$ $R = Mg - F\sin\theta$ ...(i)

Also, $R = Mg - F\sin\theta$ ...(ii)

From Eqs. (i) and (ii)

$$\mu(Mg - F\sin\theta) = F\cos\theta$$

$\Rightarrow$ $\mu Mg = \mu F\sin\theta + F\cos\theta$

$\Rightarrow$ $\mu Mg = F(\mu F\sin\theta + \cos\theta)$

$\Rightarrow$ $F = \dfrac{\mu Mg}{(\cos\theta + \mu\sin\theta)}$

43. Faraday constant (F) represents the amount of charge required to deposit or liberate one kilogram equivalent of any element.

F = (Number of ions in kilogram equivalent) × (charge of one ion)

$$= \left(\frac{N}{n}\right)(ne) = Ne$$

= Avogadro's number × charge of one electron

44. Velocity of source, $v_s = r\omega = 0.7 \times 5 = 3.5$ m/s

Velocity of sound, $v = 332$ m/s

Frequency of source, $\nu = 1000$ Hz

Maximum apparent frequency heard by the listener standing away is given by

$$\nu' = \frac{v\nu}{(v - v_s)} = \frac{332 \times 1000}{(332 - 3.5)}$$

$$= 1010 \approx 1000 \text{ Hz}$$

45. In this type of prism combination, net deviation is zero, *i.e.*,

$$\delta + \delta' = 0$$

or $(n-1)A + (n'-1)A' = 0$

or $|A'| = \dfrac{(n-1)A}{(n'-1)} = \dfrac{(1.54-1)}{(1.72-1)} \times 6°$

$= 4° 50'$

46. Power dissipated, $P = \dfrac{V^2}{R}$

If the battery is ideal then V remains constant.

$\therefore$ $P \propto \dfrac{1}{R}$

So, on increasing R, power decreases.

47. In a transformer, frequency remains unchanged.

48. When lift is accelerated in upward motion

$$w = m(g + a)$$

or $708 = m(g + a)$

$\Rightarrow$ $m = \dfrac{708}{(g + a)}$ ...(i)

When lift is accelerated downword motion

$$W = m(g - a) \text{ or } 468 = m(g - a)$$

$\Rightarrow$ $m = \dfrac{468}{(g - a)}$ ...(ii)

From Eqs. (i) and (ii), we get

$$a = 2.04 \text{ m/s}^2$$

So, the actual weight of man,

$$m = \frac{708}{(10 + 2.04)} = 58.8 \text{ kg}$$

$$= 588 \text{ N}$$

49. For an adiabatic process

$$T^{\gamma} p^{1-\gamma} = \text{constant}$$

On simplification, we get $p \propto T^{\gamma/\gamma - 1}$ ...(i)

Here, given $p \propto T^3$ ...(ii)

On comparing Eqs. (i) and (iii), we have

$$\frac{\gamma}{\gamma - 1} = 3 \Rightarrow \gamma = \frac{3}{2}$$

50. Let the frequency of first fork = n

then the frequency of last fork = $3n$

Since two successive forks given four beats.

Hence, we have

Frequency of first fork = n

Frequency of 2nd fork = $n + 4$

Frequency of 3rd fork = $n + 2 \times 4$

Frequency of 4th fork = $n + 3 \times 4$

So, the frequency of Nth fork = $n + (N - 1)4$

But Nth fork is the last fork

$\therefore$ $3n = n + (N - 1)4$

$\Rightarrow$ $2n = (N - 1)4$

Here, $N = 56$

$\Rightarrow$ $2n = (56 - 1) \times 4 = 55 \times 4 = 220$

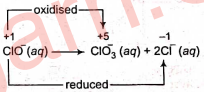
$\Rightarrow$ $n = \dfrac{220}{2} = 110$ Hz

Chemistry

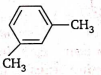
1. Gold number is defined as the minimum amount of lyophilic colloid in milligrams, which prevents the flocculation (coagulation) of 10 mL gold sol (containing 0.5 to 0.06 g of gold per litre) by the addition of 1 mL of 10% NaCl solution. More is the gold number, less is the protective power of the lyophilic colloid, since it means that the amount required is more,

Protective colloids	Gold number
Starch	15–25
Sodium oleate	0.4
Gelatin	0.005–0.01
Gum arabic	0.15

2. A reaction in which the same species is simultaneously oxidised as well as reduced is called a disproportionation reaction. For such redox reactions to occur, the reacting species must contain an element which has atleast three oxidation states. The element in the reacting species is present in the intermediate oxidation state while the higher and lower oxidation states are available for reduction and oxidation to occur.

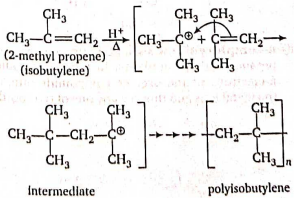


3. According to Huckel's rule, the molecules which are considered as aromatic, refer to planar cyclically conjugated structures having $(4n + 2)$ π-electrons (where n is 0, 1, 2, 3 ... etc.). The alicyclic compounds which do not have $(4n + 2)$ π-electrons are called non-aromatic compound. *e.g.*,



Aromatic molecule containing 6π electron $= (4 \times 1 + 2)\pi$ electrons.

4. 2-methylpropene or isobutylene undergoes cationic polymerisation easily in presence of BF_3 or H_2SO_4 since it has two electron donating methyl groups that will stabilize the intermediate carbocation,



5. The ease of liquefaction of a gas depends upon the magnitude of the attractive forces present in its atoms or molecules and He being smallest molecules among all the noble gases, the intermolecular forces of attraction in them are negligible.

Weaker are the intermolecular forces, more difficult it is to liquify that gas and lower would be the critical temperature of that gas. T_c for helium = 5.2 K (the temperature above which it cannot be liquified however high pressure may be applied on the gas).

That's the reason, when natural gas is compressed to about 100 atm and could to 78 K, helium does not liquify while all other gases get liquified. Thus, helium has lowest critical temperature (5.2 K) among all known gases.

6. Number of half-lives, $n = \dfrac{300}{150} = 2$, *i.e.*, $n = 2$

Amount left after 2 half-lives $= N - \dfrac{N_0}{(2)^n}$

$$= \frac{1.0}{2^2} = \frac{1.0}{4} = 0.25\text{g}$$

7. A slag is an easily fusible material which is formed when gangue still present in the roasted or the calcined are combines with the flux. For example, in the metallurgy of copper, the sulphide ore (*i.e.*, copper pyrites) is roasted in a reverberatory furnace and converted into a mixture of FeO and Cu_2O.

$$2CuFeS_2 + O_2 \xrightarrow{\Delta} Cu_2S + 2FeS + SO_2$$

Copper pyrites

$$Cu_2S + 3O_2 \xrightarrow{\Delta} 2Cu_2O + 2SO_2$$

$$2FeS + 3O_2 \xrightarrow{\Delta} 2FeO + 2SO_2$$

To remove FeO (basic impurity), an acidic flux silica is added during smelting, FeO then combines with SiO_2 to form ferrous silicate ($FeSiO_3$) slag which floats area molten matte.

$$\underset{}{FeO} + \underset{\text{flux}}{SiO_2} \longrightarrow \underset{\text{slag}}{FeSiO_3}$$

Thus, the role of silica in the matellurgy of copper is to remove iron oxide as slag.

8. Gypsum is a very soft sulphate mineral composed of calcium sulphate dihydrate, with the chemical formula $CaSO_4 \cdot 2H_2O$.

On heating, gypsum loses water and gives the hemihydrate $\left(CaSO_4 \cdot \frac{1}{2}H_2O\right)$ or the anhydrite.

$$\underset{\text{gypsum}}{2CaSO_4 \cdot 2H_2O} \xrightarrow{120°C} \underset{\substack{\text{plaster of Paris} \\ \text{(Calcium sulphate} \\ \text{hemihydrate)}}}{(CaSO_4)_2 \cdot H_2O} + 3H_2O$$

The hemihydrate is known as calcined gypsum or plaster of Paris.

9. The equation for the reaction of oxalic acid with a solution of caustic potash.

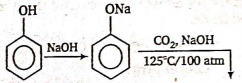
$$2KOH(aq) + H_2C_2O_4 \cdot 2H_2O(aq) \longrightarrow K_2C_2O_4(aq) + 4H_2O$$

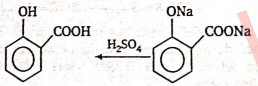
Molarity equation, $\frac{M_1V_1}{n_1} = \frac{M_2V_2}{n_2}$

where M_1 = molarity of acid = 5 millimol
M_2 = molarity of base = 5 millimol
V_1 = volume of acid = 100 mL
V_2 = volume of base = 100 mL
n_1 = stoichiometric coefficient of acid = 1
n_2 = stoichiometric coefficient of base = 2

As it is clear from balanced equation that 2 moles of KOH are required to neutralize one mole of oxalic acid. Thus, the solution formed by 5 millimoles of KOH and 5 millimoles of oxalic acid mixed and dissolved in 100 mL water will be acidic.

10. The Kolbe-Schmidt reaction or Kolbe reaction is a carboxylation chemical reaction that proceeds by heating sodium phenolate (the sodium salt of phenol) with carbon dioxide under pressure (100 atm, 125°C), then treating the product with sulphuric acid. The final product is salicylic acid, which is the precursor of aspirin (analgesic)

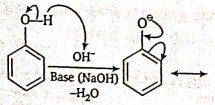


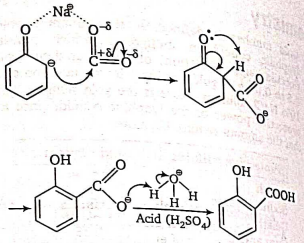


salicylic acid sodium salicylate

The Kolbe-Schmidt reaction proceeds *via* the nucleophile addition of a phenolate to carbon dioxide to give the salicylate.

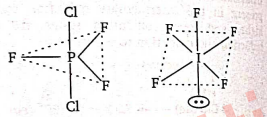
In this reaction, CO_2 act as an electrophile. The final step is reaction of salicylate with acid to form desired salicylic acid.



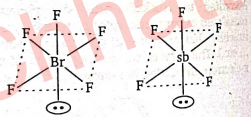


salicylic acid

11. PF_3Cl_2 has trigonal bipyramidal structure, IF_5, BrF_5 and SbF_5^{2-} have octahedral geometries with one position occupied by a lone pair of electrons (square pyramidal geometry).

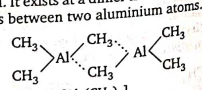


PF_3PCl_2 (sp^3d-hybridisation) IF_5 (sp^3d^2-hybridisation)



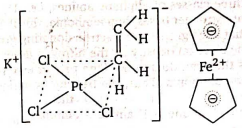
BrF_5 (sp^3d^2-hybridisation) $[SbF_5]^{-2}$ (sp^3d^2-hybridisation)

12. (i) **σ-bonded complexes** Trimethyl aluminium is a σ-bonded organometallic compound in which the metal and carbon atom of ligand are joined together with a σ-bond. It exists at a dimer and two methyl group act as bridges between two aluminium atoms.



$[Al_2(CH_3)_6]$

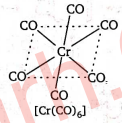
(ii) **π-complexes** Zeise's salt and ferrocene are π-bonded organometallic complexes. In these compounds, the π-electrons of the organic compounds interact with the metal ions and thus occupy one of the coordination site.



Zeise's salt Ferrocene

$K^+[PtCl_3(\eta_2 - C_2H_4)]$ $Fe(\eta^2 - C_5H_5)_2]$

(iii) **Complexes with characteristics of both σ-and π-bonding** In metal carbonyl $Cr(CO)_6$, the carbon of CO donates a pair of electrons to the metal. The metal carbon bond in metal carbonyls has σ as well as π electrons.



$[Cr(CO)_6]$

13. The equation representing the electrolysis of H_2O to form O_2 and H_2

$$2H_2O \longrightarrow \underset{\text{At cathode}}{2H_2} + \underset{\text{At anode}}{O_2}$$

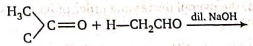
$2 \times 18 = 36$ g 2 mol 1 mol

Since, 36 g of H_2O on electrolysis produces 1 mole of O_2

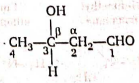
∴ 90 g of H_2O on electrolysis produces

$= \frac{1}{36} \times 90 = 2.5$ mol of O_2

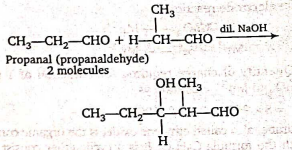
14. When two molecules of an aldehyde or a ketone condense in presence of a dilute alkali (dil. NaOH, Na_2CO_3, $Ba(OH)_2$ etc.) to form a β-hydroxyaldehyde or a β-hydroxyketone respectively, these β-hydroyaldehydes or ketones are callectively called aldols and the reaction is called aldol condensation. The aldol reaction requires an aldehyde or ketone that contains at least one α-hydrogen in order to form enol or enolate. Thus, acetaldehyde, propanaldehyde and acetone undergo aldol condensation since they contain α-hydrogen atom.



Ethanal acetaldehyde
(2 molecules)

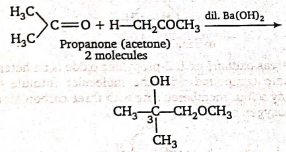


β-hydroxybutanaldehyde
(3-hydroxy butanol)



Propanal (propanaldehyde)
2 molecules

3-hydroxy-2-methylpentanol
(β-hydroxy α-methylalehyde)



Propanone (acetone)
2 molecules

β-hydroxy-β-methyl acetone

15. Shiff's nitrometre is a long U tube used to measuring the volume of N_2 in estimation of N_2 by Duma's method. It contains about 40% KOH solution and a mercury seal at it the bottom which prevents KOH solution from having sucked back into the combustion tube.

16. The order of increasing energies of the various orbitals or electrons can be calculated on the basis of $(n + l)$ rule. According to this rule, the lower the value of $(n + l)$ for an orbital, lower is its energy. However, if the two different types of orbitals have the same values of $(n + l)$, the orbital with lower value of n has lower energy.

Type of electron	value of n	value of l	value of $(n + l)$	Relative energy
e_1	4	0	4 + 0 = 4	Higher energy than e_2, because of high value of $n = 1$
e_2	3	1	3 + 1 = 4	Lower energy than e_1, because of two value of $(n = 3)$
e_3	3	2	3 + 2 = 5	Highest energy
e_4	3	0	3 + 0 = 3	Lowest energy

So, the order of energy, $e_3 > e_1 > e_2 > e_4$

17. Silver is a highly unreactive metal so it does not react with the oxygen of air easily. But air usually contains a parts per billion concentrations of pollutant gases, such as HCl, H_2S, SO_2, NO_2 and ozone. The exposure of silver to H_2S gas at 25°C and air containing 75% relative humidity produces a black coating consisting of silver sulphide (Ag_2S) on its surface.

Similarly, the exposure of silver to NO_2 gas at 25°C and air containing 75% relative humidity produces a thin film of $AgNO_3$ (transparent) and Ag_2O (black) on its surface. Thus, silver ornaments gradually turn black due to formation of layers of Ag_2S and Ag_2O.

18. The electrode reaction is

$$MnO_4^- \longrightarrow Mn^{2+}$$

i.e., $$Mn^{7+} + 5e^- \longrightarrow Mn^{2+}$$

$\therefore$ Quantity of charge required for reduction of 1 mole of MnO_4^- to Mn^{2+} = 5 faradays

$= 5 \times 96500\ C = 482500\ C$

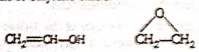
19. Oxirane, also called ethylene oxide, is the organic compound with the formula C_2H_4O. It is a cyclic ether consist of two carbon atoms and one oxygen atom, formic a ring.

oxirane (ethylene oxide)

Whereas oxetane or 1, 3-propylene oxide is an heterocyclic organic compound with the molecular formula C_3H_6O, having a four membered ring with three carbon atoms and one oxygen atom.

oxetane (1,3-propylene oxide)

Vinyl alcohol, with the formula CH_2CHOH, is an isomer of oxirane or ethylene oxide.



vinyl alcohol oxirane (ethylene oxide)

20. The stability of a nucleus depends upon the neutron to proton $\left(\frac{n}{p}\right)$ ratio in the nucleus. Thus,

(i) The lower elements (upto $Z = 20$), the stable nuclei have about equal number of protons and neutrons

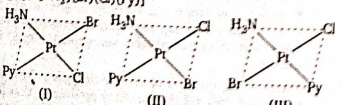
i.e., $\frac{n}{p} = 1$

e.g., $^{40}_{20}Ca$, $^{27}_{13}Al$

(ii) For higher elements to be stable, there must be more neutrons than protons *i.e.,* $\frac{n}{p} > 1$. *e.g.,* $^{55}_{25}Mn$, $^{119}_{50}Sn$

21. Isomerism which occurs due to different relative arrangements of ligands around central metal atom is known as geometrical isomerism. Complexes of formula M_{ABCD} may exist in three isomeric forms.

e.g., $[Pt(NH_3)(Br)(Cl)(Py)]$



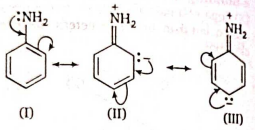
(I) (II) (III)

square planar complexes do not show optical isomerison since they are not optically active as they have all the ligands and metal atoms in one plane, that's why there is a plane of symmetry.

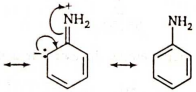
22. All the three classes of aliphatic amines, *i.e.,* 1°, 2° and 3° amines are stronger bases than ammonia, this is due to the reason that alkyl groups are electron donating groups. As a result, the electron density on the nitrogen atom increases and thus they can donate the lone pair of electrons more easily than ammonia. Thus, the basicity of amines should decrease in the order

3° amine > 2° amine > 1° amine > ammonia

On the other hand, aromatic amines are far less basic than ammonia. This is because due to resonance in aniline, the lone pair of electrons on the nitrogen atom gets delocaliged over the benzene ring and thus is less easily available for protonation.



(I) (II) (III)



(IV) (V)

resonance in aline

But when the hydrogen atom of the amino group in aniline are replaced by electron donating alkyl groups, the basicity of resultant arylamines increases. *e.g.,* N-methylaniline is a stronger base than aniline and N, N-dimethyl aniline is even stronger than N-methylaniline.

However, they are not stronger bases than ammonia. Thus, the basicity of N-substituted anilines relative to aniline follows the sequence

$$NH_3 > C_6H_5-N\begin{matrix}CH_3\\CH_3\end{matrix} > C_6H_5-N\begin{matrix}H\\CH_3\end{matrix} > C_6H_5-NH_2$$

N,N-dimethyl aniline N-methylaniline

Thus, the overall decreasing order of basicities of different amines is

$$C_2H_5-N\begin{matrix}CH_3\\CH_3\end{matrix} > C_2H_5-N\begin{matrix}H\\CH_3\end{matrix} > NH_3 >$$

N,N-dimethyl ethanamine N-methyl ethanamine Ammonia

$$C_6H_5-N\begin{matrix}CH_3\\CH_3\end{matrix} > C_6H_5-N\begin{matrix}H\\CH_3\end{matrix} > C_6H_5-NH_2$$

N,N-dimethyl aniline N-methyl aniline Aniline

23. Aluminium is extracted by electrolysis of fused or molten alumina (Al_2O_3) containing cryolite (Na_3AlF_6) added to lower the melting point of alumina to around 1140 K and to enhance its conductance, as fused alumina is a bad conductor of electricity.

(Hall and Heroult process).

24. For the reaction,

$$2NO + Br_2 \longrightarrow 2NOBr$$

the following mechanism has been given

$$NO + Br_2 \xrightarrow{Fast} NOBr_2 \quad ...(i)$$

$$NOBr_2 + NO \xrightarrow{Slow} 2NOBr \quad ...(ii)$$

As the second step is the rate determining step,

$$Rate = k'[NOBr_2][NO] \quad ...(iii)$$

From (i) step

$$k'' = \frac{[NOBr_2]}{[NO][Br_2]}$$

or $$[NOBr_2] = k''[NO][Br_2] \quad ...(iv)$$

By substituting this value in rate law step (iii), we get

$$Rate = k'k''[NO]^2[Br_2]$$

or $$Rate = k''[NO]^2[Br_2]$$

25. Electron affinity of an element is equal to the energy released when an electron is added to valence shell of an isolated gaseous atoms. For example, when an electron is added to oxygen atom to form O^- ion, energy is released, *i.e.,* EA_1, value is exoergic or negative.

$$O(g) + e^-(g) \longrightarrow O^-(g)$$

$$\Delta H(EA_1) = -141\ kJ\ mol^{-1}$$

(Energy is released)

But the addition of second electron to O^- ion to form O^{2-} ion is more difficult and energy is needed to overpower the repulsion forces between negatively charged atomic sphere and test electrons. Thus, the EA_2 value of oxygen atom is positive or endoergic.

$$O^-(g) + e^-(g) \longrightarrow O^{2-}(g)$$

$$\Delta H(EA_2) = +780\ kJ\ mol^{-1}$$

(Energy is absorbed.)

Thus, the EA_1 values for all elements are exoergic (however for some elements endoergic, *e.g.,* noble gases and alkali earth metals) while rest all (EA_2, EA_3...) endoergic.

26. When a solid substance changes into its liquid state at its melting point, the process takes place by absorption of heat from the surroundings.

e.g., melting of ice

$$H_2O(s) \longrightarrow H_2O(l) \quad \Delta H^0_{fus} = +6.0\ kJ\ mol^{-1}$$

Thus, enthalpy change for this process is positive or there is an increase in enthalpy, but still the reaction is spontaneous, because the liquid state of water is more random than its solid state. Hence, it may be concluded that melting of ice is accompanied by an increase of entropy.

$$\Delta G = \Delta H - T\Delta S$$

To sum up, a reaction to be spontaneous, the value of ΔG, *i.e.,* the Gibbs' free energy change must be negative or there must be a decrease in free energy, thus, ΔH must be smaller than $T\Delta S$.

27. α-hydrogen of a carboxylic acid can be replaced by halogen (chlorine or bromine) using red phosphorus as catalyst to afford an α-halocarboxylic acid. This reaction is called Hell-Volhard-Zelinsky reaction.

$$R-CH_2-COOHX_2 \xrightarrow{Red\ P} R-\underset{X}{\underset{|}{CH}}-COOH + HX$$

carboxylic acid α-halocarboxylic acid

(where, x = Cl, Br)

28. The determination of age of minerals and rocks is an important part of ancient geological studies. The age of rock can be estimated by uranium–lead dating.

Suppose x g of $^{206}_{82}Pb$ is found with y g of $^{238}_{92}U$.

$\lambda \rightarrow$ decay constant of uranium (known)

$$^{238}_{92}U \longrightarrow {}^{206}_{82}Pb$$

Amount of $^{238}_{92}U$ disintegrated $= \frac{x}{206} \times 238$ in time t

Hence initial amount of $^{238}_{92}U$

$$= y + \frac{x}{206} \times 238$$

$$\lambda t = 2.303 \log\left(\frac{N_0}{N}\right)$$

where, $N_0 = y + \frac{x}{206} \times 238$

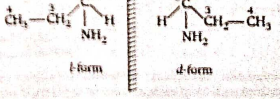
and $N = y$

$$\therefore \quad t = \frac{2.303}{\lambda} \log\left[\frac{y + \frac{x}{206} \times 238}{y}\right]$$

where, t = age of rock

On the other hand, the age of minerals is determined by helium dating and the age of animals and plants died, *i.e.,* fossils is determined by radiocarbon dating.

29. Among the different isomers of $C_4H_{11}N$, a primary amine *i.e.,* butan-2-amine is an optically active compound, which contains a chiral C-atom *i.e.,* a carbon atom which is linked to four different groups.



l-form d-form

enantiomers of butan-2-amine
(1° amine)

30. The electronic configuration $1s^2 2s^2 2p^6$ belongs to noble gas neon. All the noble gases are monoatomic in nature because they have complete valence shell and stable electronic configuration (ns^2np^6) and so they are chemically inert and do not enter in chemical combination.

$C = Ne_{(10)} = 1s^2 2s^2 2p^6$

31. A buffer solution can be described as a solution, which will resist changes in pH when a small amount of a strong acid or base is added.

An acidic buffer is compound of a weak acid and its conjugated base or a weak acid and the salt of the weak acid and a strong base. e.g.,

$(CH_3COOH + CH_3COONa)\ CH_3COOH \rightleftharpoons CH_3COO^- + H^+$.

(feebly ionised)

$CH_3COONa \rightleftharpoons CH_3COO^- + Na^+$

(completely ionised)

How to make a acidic buffer

(i) Start with taking weak acid (CH_3COOH) and strong base (NaOH) in 2 : 1 molar concentration ratio, so that a sufficient number of moles of the CH_3COOH nutralize completely the same number of moles of NaOH.

$$\underset{1\ mol}{CH_3COOH} + \underset{1\ mol}{NaOH} \longrightarrow \underset{\text{Sodium acetate}\ 1\ mol}{CH_3COONa} + H_2O$$

How to make a acidic buffer

Now, the resulting solution will contain the salt and acid in equimolar concentration and water.

$$\underset{\text{1 limiting (mol) (excess 2 mol)}}{OH^-(aq)} + H^+(aq) \longrightarrow \underset{\text{salt (1 mol) buffer solution}}{A^-(aq)} + \underset{\text{remaining (mol)}}{H^+(aq)}$$

In the buffer solution, $[HA] = [A^-]$

i.e., equimolar concentrations

(ii) The acidic buffer can also be prepared by taking equimolar concentrations and volume of weak acid (CH_3COOH) and the salt of the weak acid and strong base (CH_3COONa).

32. The hydrogen atoms attached to the triple bond of the alkynes, *i.e.*, acetylenic hydrogens are acidic in nature and known as active hydrogen.

This acidic property is shown by terminal alkynes or 1-alkynes only (alkynes in which the triple bond is at the end of the chain). The reaction with Na in liquid NH_3 is considered as test of acidity of terminal alkynes.

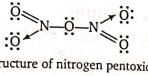
$$\underset{\text{But-1-yne}}{CH_3-CH_2-C\equiv CH} + Na \xrightarrow[\Delta]{Liq.\ NH_3} \underset{\text{sodium but-1-ylide}}{CH_3-CH_2-C\equiv C^-Na^+}$$

The reason of acidity of terminal alkynes is greater electronegativity of carbon atom of C—H triple bond, which is *sp*-hybridised. In other words, electrons of C—H bond are displaced more towards the carbon than towards the hydrogen atom, hence it can be removed as a proton (H^+) by a strong base. Consequently, alkynes behave as acids.

33. The Kjeldahl bulb or Kjeldahl trap between the digestion flask (Kjeldahl flask) and the condenser improves distillation efficiency by preventing carryover of the alkaline digestion mixture and water vapours into the receiving flask and the large lower tube facilitates the return of any condensate to the digestion flask. The slightest bit of contaminated of the receiving solution can cause significant error in the titration step.

The condensate in the bulb/trap is maintained above pH 7 by the non-volatile alkali sucrubbed from the vapour and it is kept at or near the boiling point of passing steam. Under these conditions, no ammonia should be retained in the condensate of the Kjeldahl's trap.

34. Nitrogen pentoxide (N_2O_5) has both covalent as well as coordinate bonds. In the gaseous state, it exists as a symmetrical molecule having the structure $O_2N—O—NO_2$. The N—O—N bond is almost linear. X-ray studies reveals the ionic nature of solid N_2O_5 *i.e.*, nitronium nitrate, $Na_2^+NO_3^-$.



Structure of nitrogen pentoxide

35. Hydration represents the dissolution of a substance in water by absorbing water molecules by weak valency force. The energy released when 1 g mol of an ion in the gaseous state is dissolved in water to get it hydrated is called hydration energy.

$M^+(g) + aq \longrightarrow M^+(aq)\quad \Delta H = -$ Hydration energy

(i) Smaller the cation, greater is the degree of hydration

$Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

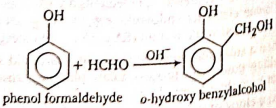
$Be^{2+} > Mg^{2+} > Ca^{2+} > Sr^{2+} > Ba^{2+}$

(ii) The hydration energy of ions increases with increase in their valencies.

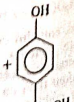
$Al^{3+} > Mg^{2+} > Na^+$

Thus, Mg^{2+} ion has higher hydration energy than Na^+ ion, but lower hydration energy than Al^{3+} and Be^{2+}.

36. Phenol formaldehyde polymers are the oldest synthetic polymers. These are obtained by condensation of phenol with formaldehyde in the presence of either an acid or a base catalyst.

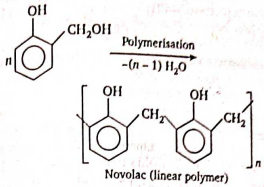


phenol formaldehyde *o*-hydroxy benzylalcohol



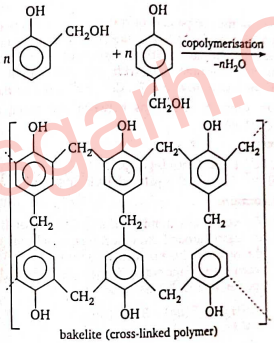
p-hydroxy benzylalcohol

The condensation of *o*-hydroxybenzyl alcohol or *p*-hydrony benzylalcohol gives a linear polymer.



Novolac (linear polymer)

The *o*-and *p*-substituted phenols can undergo cross linkage to form an infusible solid called bakelite.



bakelite (cross-linked polymer)

37. Aluminium hydroxide dissociates according to the equations

$Al(OH)_3(aq) \rightleftharpoons Al(OH)_3(aq)$

$\rightleftharpoons Al^{3+}(aq) + 3OH^-(aq)$

Let the solubility of $Al(OH)_3 = S$ mol L^{-1}

$\because\ Al(OH)_3(aq) \rightleftharpoons \underset{S}{Al^{3+}(aq)} + \underset{3S}{3OH^-(aq)}$

$\because\quad K_{sp} = [Al^{3+}][OH^-]^3$

$\because\quad K_{sp} = (s)(3s)^3 = 27s^4$

hence, $K_{sp} = 27s^4$

38. The stability of a complex is measured as stability constant, $K_{stability}$

(A) $[AlF_6]^{3-}$; $K_{stability} = 2.5 \times 10^4$

(B) $[Cd(CN)_4]^{2-}$; $K_{stability} = 1.7 \times 10^{16}$

(C) $[Ag(CN)_2]^-$; $K_{stability} = 6.6 \times 10^{18}$

(D) $[Zn(CN)_4]^{2-}$; $K_{stability} = 1.0 \times 10^{19}$

Thus, the increasing order of stability

$A < B < D < C$

39. Electronic configuratrions of metal ions

(i) $Cu^+ = [Ar]\ 3d^{10} 4s^0$

(ii) $Zn^{2+} = [Ar]\ 3d^{10} 4s^0$

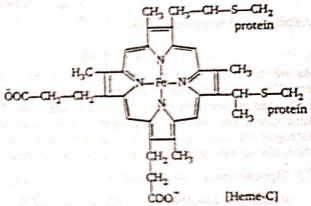
(iii) $Sc^{3+} = [Ar] 3d^0 4s^0$

(iv) $V^{4+} = [Ar] 3d^1 4s^0$

As it is clear from the electronic configuration of metal ions that V^{4+} has one unpaired electron in its *d*-subshell. The *d*-subshells are non-degenerated in presence of ligands. On exposure to visible light, the excitation of unpaired *d*-electrons takes place from lower to higher energy *d*-sublevels.

The d–d excitation during complex formation permits the absorbance of required wavelength and rest light is transmitted out. Thus, transition metals or metal ions having unpaired *d*-electrons appears coloured.

40. Heme is the prosthetic group of hemoglobin, myoglobin and cytochromes. It is an asymmetric molecule.



[Heme-C]

The heme ring system is synthesised from glycine and succinyl —CoA. It begins with condensation of glycine and succinyl —CoA with decarboxylation to form δ-aminolurilinic acid (ALA).

41. The relationship between atomic radius, *r* and the edge (*a*) of the unit cell of a cubic crystal of BCC crystal

$$r = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{4} \times 4.29\ Å$$

$$= \frac{1.732 \times 4.29}{4} = 1.857\ Å = 1.86\ Å$$

42. The magnitude of the standard electrode potential is a measure of the tendency of the half reaction to occur in the forward direction, *i.e.*, in the direction of reduction. Thus, if any element exists in more than one oxidation state, their relative stabilities can be known from the standard electrode potential data. For example in case of copper, we have

$A^{2+} + 2e^- \longrightarrow A;\ E^0 = +0.34\ V$

$A^+ + 2e^- \longrightarrow A;\ E^0 = +0.52\ V$

Thus, Cu^+ is reduced more easily hence less stable than Cu^{2+}. This is because although second ionisaton enthalpy of copper is large but enthalpy of hydration for $Cu^{2+}(aq)$ is much more negative than that for $Cu^+(aq)$ and hence it more than compensates for the second ionisation enthalpy of copper.

43. The enthalpy of neutralization is defined as the heat envolved when 1 g equivalent of an acid is neutralized by 1 g equivalent of a base or *vice-versa* in dilute solution. This is constant and its value is –57.1 kJ for neutralization of any strong acid by a strong base since in dilute solution they completely dissociate into ions.

$$H^+(aq) + OH^-(aq) \longrightarrow H_2O(l);$$
$$\Delta H_{neu} = -57.1 \text{ kJ mol}^{-1}$$

Thus, neutralization involves combination of 1 mole of H^+ ions with 1 mole of OH^- ions to form 1 mole of H_2O. Now, it is clear that 1 g equivalent (or 1 mole) of any acid on complete dissociation gives 1 mole of H^+ ions. But this is not true in case of dibasic or diprotic acid, for example, 1 mole of H_2SO_4 gives 2 moles of H^+ ions on complete dissociation. However, 1g equivalent of H_2SO_4 (= 0.5 mol) gives 1 mole of H^+ ions.

$$H_2SO_4 + 2NaOH \longrightarrow Na_2SO_4 + 2H_2O$$
$$\Delta H_{neu} = -114.64 \text{ kJ}$$

$$\therefore \text{Enthalpy of neutralization} = \frac{-114.64}{2}$$
$$= -57.32 \text{ kJ}$$

44. As we move from left to right in a period, the ionisation enthalpy increases with increasing atomic number due to increased nuclear charge and smaller atomic radii. But the first ionisation enthalpy of oxygen is lower than that of nitrogen although the nuclear charge of oxygen is higher than that of nitrogen. This is due to following reasons.

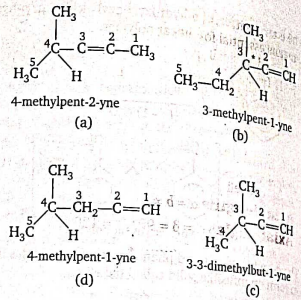
(i) The electronic configuration of N $(1s^2 2s^2 2p_x^1 2p_y^1 2p_z^1)$ in which the 2*p*-orbitals are exactly half-filled is more stable than the electronic configuration of O $(1s^2 2s^2 2p_x^2 2p_y^1 2p_y^1)$ in which the 2*p*-orbitals are neither half-filled nor completely filled. Therefore, it is difficult to remove an electron from N than from O.

(ii) The removal of an electron from O gives a stable electronic configuration with exactly half-filled 2*p* subshell, *i.e.*, $O^+(1s^2 2s^2 2p_x^1 2p_y^1 2p_z^1)$ while this is not so in case of N, *i.e.*,

$N^+(1s^2 2s^2 2p_x^1 2p_y^1 2p_z^0)$. Therefore, the first ionisation enthalpy of O is lower than that of N.

- For a compound to be optically active it must be

(i) Chiral and non-superimposable on its mirror image. The molecules which are not superimposable on their mirror images are called chiral molecules. Chiral molecule has a carbon atom linked to four different groups and this carbon atom is called as chiral carbon atom. Among all the alkynes having five membered chain one is optically active.



46. Most of the compounds of transition metals are coloured in solid as well as solution state. Colour in transition metal ions is usually due to electron transition within the *d*-shell. Intensely coloured ions with the metal in its highest oxidaton state (*e.g.*, Mn^{7+}, Cr^{6+}, Fe^{6+}) derive the colour from electrons transitions between the metal and the oxygen atom.

Iron

(i) Fe^{2+} (ferrous ion) is green.
(ii) Fe^{3+} (ferric ion) is pale yellow in colour.

Chromium

(i) The colour of chromic ion, *i.e.*, Cr^{3+} depends around the ligand around the ion. Aqueous solution contains the violet octahedral hexaaquachromium (III) ion, but when some of the water ligands are replaced by other species, such Cr^{3+} ions are green.

$$\underset{\text{Violet, octahedral}}{[Cr(H_2O)_6]^{3+}(aq)} + 3OH^-(aq) \longrightarrow \underset{\text{Green, octahedral}}{[Cr(OH)_3(H_2O)_6](s)} + 3H_2O(l)$$

(ii) In chromate ion (CrO_4^{2-}), Cr occurs in +6 oxidation state and it is yellow in colour.

47. For the given reaction

$$CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(s)$$

$$\Delta n_g = n_{products} - n_{reactants}$$
$$= 1 - 1.5$$
$$= -0.5$$

Hence, $K_p = K_c(RT)^{\Delta n}$

or $\dfrac{K_p}{K_c} = (RT)^{-0.5} = \dfrac{1}{\sqrt{RT}}$

48. The term vitamins was coined, by C Funk in 1912, which means essential for life. Vitamins are accessory fast factors, required in small quantity for controlling metabolism and body functioning, *e.g.*, they help keeping our eyes, bones, teeth and gums healthy, but do not provide energy.

Vitamins are an essential component of our diet, because they are not produced inside the body and in the absence of vitamins, a number of chemical reactions cannot take place.

49. A hexagonal crystal system has the following parametres of its unit cell

Axial lengths $a = b \neq c$

Axial angles $\alpha = \beta = 90°$, $r = 120°$

Mathematics

1. Given, diagonal of parallelogram are

$$d_1 = 3i + j - 2k$$
and $$d_2 = i - 3j + 4k.$$

Area of parallelogram $= \frac{1}{2}|d_1 \times d_2|$

$$= \frac{1}{2}\begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$
$$= \frac{1}{2}|i(4-6) - j(12+2) + k(-9-1)|$$
$$= \frac{1}{2}|-2i - 14j - 10k|$$
$$= |-i - 7j - 5k|$$
$$= \sqrt{(-1)^2 + (-7)^2 + (-5)^2}$$
$$= \sqrt{1 + 49 + 25} = \sqrt{75}$$
$$= 5\sqrt{3} \text{ sq units}$$

2. $(i + j)\cdot[(j + k) \times (k + i)]$

$$= (i + j)\cdot\begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$
$$= (i + j)\cdot[i(1-0) - j(0-1) + k(0-1)]$$
$$= (i + j)\cdot(i + j - k)$$
$$= 1 + 1 - 0 = 2$$

3. Given, differential equation is

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

e.g., cinnabar (HgS), ICl, Graphite, ZnO, PbI_2 etc.

50. By de-Broglie equation

$$\lambda_A = \frac{h}{m_A v_A}$$
and $$\lambda_B = \frac{h}{m_B v_B}$$
$$\therefore \frac{\lambda_A}{\lambda_B} = \frac{m_B v_B}{m_A v_A} \quad \text{...(i)}$$

As $m_B = 25\%$ of mass of A
$\therefore$ $m_A = 1$
and $m_B = 0.25$
similarly, $v_B = 75\%$ of velocity of A
$\therefore$ $v_A = 1$ and $v_B = 0.75$

By putting the values in Eq. (i)

$$\frac{\lambda_A}{\lambda_B} = \frac{0.25 m_A \times 0.75 v_A}{m_A \times v_A}$$
$$\Rightarrow \frac{\lambda_A}{\lambda_B} = 0.1875$$
$$\Rightarrow \lambda_B = \frac{\lambda_A}{0.1875} = 5.33 \text{ Å}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right) \quad \text{...(i)}$$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \text{ in Eq. (i),}$$

we get

$$v + x\frac{dv}{dx} = v - \tan v$$
$$\Rightarrow x\frac{dv}{dx} = v - \tan v - v = -\tan v$$
$$\Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x}$$
$$\Rightarrow \frac{\cos v}{\sin v}dv = \frac{-dx}{x}$$

On integrating both sides, we get

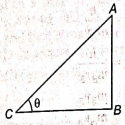
$$\int \frac{\cos v}{\sin v}dv = -\int \frac{dx}{x}$$
$$\Rightarrow \log(\sin v) = -\log x + \log C$$
$$\Rightarrow \log(\sin v) + \log x = \log C$$
$$\Rightarrow \log(x \sin v) = \log C \quad (\because \log m + \log n = \log mn)$$
$$\Rightarrow x \sin v = C$$

On putting $v = \frac{y}{x}$, we get

$$x\sin\left(\frac{y}{x}\right) = C$$

which is the required solution.

4. Let AB be the pole and its shadow is BC. According to the question,



Shadow of the pole $= \sqrt{3}$ height of the pole

$\Rightarrow \quad BC = \sqrt{3}AB$ ...(i)

Now, let angle of elevation of Sun is θ.

Then, in ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$\Rightarrow \quad \tan\theta = \frac{AB}{\sqrt{3}AB}$ [from Eq. (i)]

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore \quad \theta = 30°$

5. Given, curves are $c_1 \equiv xy = k$ ...(i)

and $c_2 \equiv x = y^2$ ...(ii)

On solving Eqs. (i) and (ii), we get the intersection point $(k^{2/3}, k^{1/3})$.

On differentiating Eqs. (i) and (ii) w.r.t. x, we get

for Eq. (i), $x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{c_1} = \frac{-y}{x}$ ...(iii)

for Eq. (ii),

$$1 = 2y\frac{dy}{dx}$$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{c_2} = \frac{1}{2y}$ ...(iv)

At point $(k^{2/3}, k^{1/3})$, $\left(\frac{dy}{dx}\right)_{c_1} = -\frac{k^{1/3}}{k^{2/3}}$

and $\left(\frac{dy}{dx}\right)_{c_2} = \frac{1}{2k^{1/3}}$

Since, the curves cut at right angle, then

$$\left(\frac{dy}{dx}\right)_{c_1} \cdot \left(\frac{dy}{dx}\right)_{c_2} = -1$$

$\Rightarrow \quad \left(-\frac{k^{1/3}}{k^{2/3}}\right)\left(\frac{1}{2k^{1/3}}\right) = -1$

$\Rightarrow \quad k^{2/3} = \frac{1}{2}$

$\therefore \quad k^2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

6. Given, $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are two unit vectors.

$\therefore \quad |\hat{\mathbf{a}}| = |\hat{\mathbf{b}}| = 1$

Now, $|\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2 = |\hat{\mathbf{a}}|^2 + |\hat{\mathbf{b}}|^2 - 2|\hat{\mathbf{a}}||\hat{\mathbf{b}}| \cdot \cos\theta$

$\Rightarrow \quad |\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2 = 1 + 1 - 2\cos\theta$

$\Rightarrow \quad |\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2 = 2(1 - \cos\theta)$

$\Rightarrow \quad \frac{|\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2}{2} = 1 - 1 + 2\sin^2\frac{\theta}{2}$

$\Rightarrow \quad \frac{|\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2}{2} = 2\sin^2\frac{\theta}{2}$

$\Rightarrow \quad \sin^2\frac{\theta}{2} = \frac{|\hat{\mathbf{a}} - \hat{\mathbf{b}}|^2}{4}$

Taking square root on both sides, we get

$$\sin\frac{\theta}{2} = \frac{|\hat{\mathbf{a}} - \hat{\mathbf{b}}|}{2}$$

7. Given, α and β are the roots of $ax^2 + bx + c = 0$.

$\therefore \quad a\alpha^2 + b\alpha + c = 0$ ...(i)

and $\alpha\beta^2 + b\beta + c = 0$ ...(ii)

Also, $\alpha + \beta = -\frac{b}{a}$ ...(iii)

and $\alpha\beta = \frac{c}{a}$ ...(iv)

Now, $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to \alpha} \frac{0 + \sin(ax^2 + bx + c) \cdot (2ax + b)}{2(x - \alpha)}$ (by L-Hospital rule)

$= \lim_{x \to \alpha} \frac{(2ax + b)\sin(ax^2 + bx + c)}{2(x - \alpha)}$ $\left(\frac{0}{0} \text{ form}\right)$

$(2ax + b)\cos(ax^2 + bx + c) \cdot$

$= \lim_{x \to \alpha} \frac{(2ax + b) + 2a\sin(ax^2 + bx + c)}{2(1 - 0)}$

$= \frac{1}{2}(2a\alpha + b)\cos(a\alpha^2 + b\alpha + c)(2a\alpha + b)$

$+ 2a\sin(a\alpha^2 + b\alpha + c)$

$= \frac{1}{2}(2a\alpha + b)^2\cos(0) + 2a\sin(0)$ [using Eq. (i)]

$= \frac{(2a\alpha + b)^2}{2} = \frac{a^2}{2}\left(2\alpha + \frac{b}{a}\right)^2$

$= \frac{a^2}{2}[2\alpha - (\alpha + \beta)]^2$ [using Eq. (ii)]

$= \frac{a^2}{2}(\alpha - \beta)^2$

8. Given, differential equation is

$$\cos\left(\frac{dy}{dx}\right) = a$$

$\Rightarrow \quad \frac{dy}{dx} = \cos^{-1} a$

$\Rightarrow \quad dy = \cos^{-1} a\,dx$

On integrating both sides, we get

$$\int dy = \cos^{-1} a \int dx + C$$

$\Rightarrow \quad y = \cos^{-1} ax + C$ ...(i)

When $x = 0$, then $y = 2$

Then, from Eq. (i), we get

$$2 = 0 + C \Rightarrow C = 2$$

On putting the value of C in Eq. (i), we get

$$y = x\cos^{-1} a + 2$$

$\Rightarrow \quad y - 2 = x\cos^{-1} a$

$\Rightarrow \quad \frac{y - 2}{x} = \cos^{-1} a$

$\Rightarrow \quad \cos\left(\frac{y - 2}{x}\right) = a$

which is the required solution.

9. Let $(a + ib)^2 = 2i = 0 + 2i$

$\Rightarrow \quad a^2 - b^2 = 0$ and $2ab = 2$

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$= 0 + (2)^2 = 4$

$\Rightarrow \quad a^2 + b^2 = 2$

$\Rightarrow \quad a = \pm 1$ and $b = \pm 1$

Hence, square root of $2i = \pm(1 + i)$

10. Given, $\left|\frac{z - 1}{z + 1}\right| = 1$

On putting $z = x + iy$, we get

$$\left|\frac{x + iy - 1}{x + iy + 1}\right| = 1$$

$\Rightarrow \quad \frac{|(x - 1) + iy|}{|(x + 1) + iy|} = 1$

$\Rightarrow \quad |(x - 1) + iy| = |(x + 1) + iy|$

$\Rightarrow \quad \sqrt{(x - 1)^2 + y^2} = \sqrt{(x + 1)^2 + y^2}$

On squaring both sides, we get

$$(x - 1)^2 + y^2 = (x + 1)^2 + y^2$$

$\Rightarrow \quad x^2 + 1 - 2x + y^2 = x^2 + 1 + 2x + y^2$

$\Rightarrow \quad 4x = 0$

$\Rightarrow \quad x = 0 \Rightarrow y$-axis

which represent a straight line.

11. Given, $\sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\theta = \sin\frac{\pi}{3}$

$\Rightarrow \quad \theta = n\pi + (-1)^n\frac{\pi}{3}$

which represent the general value of θ.

12. Given, equation is $2x - \log_{10} x = 7$.

Let $f(x) = 2x - \log_{10} x - 7 = 0$

Then, at $x = 3.5$, $f(3.5) = -\log_{10} 3.5 < 0$

At $x = 4$, $f(4) = 1 - \log_{10} 4 > 0$

Hence, roots lies between 3.5 and 4.

13. Given, coefficient of correlation between x and y, $r = 0.8$

Regression coefficient of y an x, $b_{yx} = 0.2$

We know that, $r = \sqrt{b_{xy} \cdot b_{yx}}$

$\Rightarrow \quad 0.8 = \sqrt{b_{xy} \cdot 0.2}$

On squaring both sides, we get

$$(0.8)^2 = b_{xy} \cdot 0.2$$

$\Rightarrow \quad b_{xy} = \frac{0.64}{0.2}$

$\Rightarrow \quad b_{xy} = 3.2$

Hence, regression coefficient of x on y, $b_{xy} = 3.2$

14. Given, $\sin(x + y) = \log(x + y)$

On differentiating w.r.t. x, we get

$$\cos(x + y)\left(1 + \frac{dy}{dx}\right) = \frac{1}{(x + y)} \cdot \left(1 + \frac{dy}{dx}\right)$$

$\Rightarrow \cos(x + y) + \cos(x + y)\frac{dy}{dx} = \frac{1}{(x + y)} + \frac{1}{(x + y)}\frac{dy}{dx}$

$\Rightarrow \left\{\cos(x + y) - \frac{1}{(x + y)}\right\}\frac{dy}{dx} = \frac{1}{(x + y)} - \cos(x + y)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\frac{1}{(x + y)} - \cos(x + y)}{\cos(x + y) - \frac{1}{(x + y)}}$

$\Rightarrow \quad \frac{dy}{dx} = -1$

15. A conic section represents a circle, if its eccentricity e is 0.

16. $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1 + x^2}$

17. $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= \tan 9° - \tan 27° - \tan(90° - 27°) + \tan(90° - 9°)$

$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$

$= \frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°} - \frac{\sin 27°}{\cos 27°} - \frac{\cos 27°}{\sin 27°}$

$= \frac{\sin^2 9° + \cos^2 9°}{\cos 9° \cdot \sin 9°} - \frac{\sin^2 27° + \cos^2 27°}{\cos 27° \cdot \sin 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$ $(\because 2\sin A \cdot \cos A = \sin 2A)$

$= 2\left(\frac{\sin 54° - \sin 18°}{\sin 18° \cdot \sin 54°}\right)$

$= 2\left[\frac{2\cos\left(\frac{54° + 18°}{2}\right) \cdot \sin\left(\frac{54° - 18°}{2}\right)}{\sin 18° \cdot \sin 54°}\right]$

$= 4\left(\frac{\cos 36° \cdot \sin 18°}{\sin 18° \cdot \sin 54°}\right)$

$= 4\left\{\frac{\cos(90° - 54°)}{\sin 54°}\right\}$

$= 4\left(\frac{\sin 54°}{\sin 54°}\right) = 4$

18. Given, series is

$105 + 103 + 101 + \ldots + 49 + 47$

which is an AP.

Here, $a = 105$

and $d = 103 - 105 = -2$

Let nth term $= 47$

$\Rightarrow \quad a + (n-1)d = 47$

$\Rightarrow \quad 105 + (n-1)(-2) = 47$

$\Rightarrow \quad -2(n-1) = 47 - 105$

$\Rightarrow \quad n - 1 = \frac{-58}{-2}$

$\Rightarrow \quad n = 29 + 1$

$\Rightarrow \quad n = 30$

Hence, number of terms in given series is 30.

19. Given, variance of x, $\sigma_x^2 = 9$

$\Rightarrow \quad \sigma_x = \sqrt{9} = 3$

Variance of y, $\sigma_y^2 = 16$

$\Rightarrow \quad \sigma_y = \sqrt{16} = 4$

Covariance, $\text{cov}(x, y) = 8$

Now, coefficient of correlation,

$$r_{xy} = \frac{\text{cov}(x, y)}{\sigma_x \cdot \sigma_y} = \frac{8}{3 \times 4} = \frac{2}{3}$$

20. Given, $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$= 1(0-0) - 0(0-1) + 1(0-1)$

$= 0 - 0 - 1$

$\Rightarrow \quad |A| \neq 0$

Hence, A is a non-singular matrix.

21. Let yz-plane divides the line joining the points $A(3, 1, -5)$ and $B(1, 4, -6)$ in the ratio $k : 1$.

In yz-plane, x-coordinate $= 0$

$\therefore \quad \frac{k+3}{k+1} = 0$

$\Rightarrow \quad k + 3 = 0$

$\Rightarrow \quad k = -3$

Hence, the required ratio is $-3 : 1$.

22. Given, ${}^nP_5 = 20\,{}^nP_3$

$\Rightarrow \quad \frac{n!}{(n-5)!} = 20 \cdot \frac{n!}{(n-3)!}$

$\Rightarrow \quad (n-3)! = 20(n-5)!$

$\Rightarrow \quad (n-3)(n-4)(n-5)! = 20(n-5)!$

$\Rightarrow \quad n^2 - 4n - 3n + 12 = 20$

$\Rightarrow \quad n^2 - 7n + 12 - 20 = 0$

$\Rightarrow \quad n^2 - 7n - 8 = 0$

$\Rightarrow \quad n^2 - 8n + n - 8 = 0$

$\Rightarrow \quad n(n-8) + 1(n-8) = 0$

$\Rightarrow \quad (n-8)(n+1) = 0$

$\Rightarrow \quad n - 8 = 0$

or $\quad n + 1 = 0$

$\Rightarrow \quad n = 8$

or $\quad n = -1$

Since, n cannot be negative.

$\therefore \quad n = 8$

23. Given, differential equation is

$$\frac{d^2y}{dx^2} = \left\{y + \left(\frac{dy}{dx}\right)^2\right\}^{1/4}$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^4 = y + \left(\frac{dy}{dx}\right)^2$$

Hence, order $= 2$ and degree $= 4$

24. The equation of family of circles touching the y-axis at the origin is,

$$(x \pm h)^2 + y^2 = h^2$$

$$\Rightarrow \quad x^2 + y^2 + 2xh = 0 \quad \ldots(i)$$

On differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} + 2h = 0$$

or $\quad 2x + 2y \cdot y' + 2h = 0 \quad \left(\because -1\frac{dy}{dx} = y'\right)$

$\Rightarrow \quad 2h = -(2x + 2yy')$

On putting the value of $2h$ in Eq. (i), we get

$$x^2 + y^2 - x(2x + 2yy') = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2x^2 - 2xyy' = 0$$

$$\Rightarrow \quad 2xyy' + x^2 = y^2$$

which is the required differential equation.

25. Let $I = \int \frac{dx}{x^4 - 1} = \int \frac{dx}{(x^2-1)(x^2+1)}$

On apply partial fractions, we get

$$I = \frac{1}{2}\int \frac{1}{x^2-1}dx - \frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$= \frac{1}{2} \cdot \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1} \cdot x + C$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1} x + C$$

26. Given, points are $(k, 2-2k)$, $(-k+1, 2k)$ and $(-4-k, 6-2k)$. These points are collinear, then area of $\Delta = 0$

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad k(2k - 6 + 2k) - (2 - 2k)$

$(-k + 1 + 4 + k) + 1[(-k+1)(6-2k) - 2k(-4-k)] = 0$

$\Rightarrow 4k^2 - 6k - (2 - 2k)(5) + 1$

$(-6k + 2k^2 + 6 - 2k + 8k + 2k^2) = 0$

$\Rightarrow \quad 4k^2 - 6k - 10 + 10k + 4k^2 + 6 = 0$

$\Rightarrow \quad 8k^2 + 4k - 4 = 0$

$\Rightarrow \quad 2k^2 + k - 1 = 0$

$\therefore \quad k = \frac{-1 \pm \sqrt{1 + 4 \times 2}}{2 \times 2}$

$= \frac{-1 \pm 3}{4}$

$k = \frac{1}{2}, -1$

27. The direction cosine of a line parallel to x-axis are $(1, 0, 0)$.

$\therefore$ The equation of line parallel to x-axis is,

$$\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$$

28. Given, $\log_{10^4} x = y$

$\Rightarrow \quad \frac{1}{4}\log_{10} x = y \quad \left(\because \log_{\alpha^\beta} x = \frac{1}{\beta}\log_\alpha x\right)$

$\Rightarrow \quad \log_{10} x = 4y \quad \ldots(i)$

Now, $\log_{10^8} x^4 = \frac{4}{8}\log_{10} x \quad \left(\because \log_{\alpha^\beta} x^n = \frac{n}{\beta}\log_\alpha x\right)$

$= \frac{1}{2}\log_{10} x$

$= \frac{1}{2} \cdot 4y \quad$ [from Eq. (i)]

$= 2y$

29. Given, lines are $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

Shortest distance between the lines,

$$d = \left|\frac{\begin{vmatrix} -1-3 & -1-5 & -1-7 \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}}{\sqrt{(-2+6)^2 + (7-1)^2 + (-6+14)^2}}\right|$$

$$= \left|\frac{\begin{vmatrix} -4 & -6 & -8 \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}}{\sqrt{(4)^2 + (6)^2 + (8)^2}}\right|$$

$$= \left|\frac{-4(-2+6) + 6(1-7) - 8(-6+14)}{\sqrt{16 + 36 + 64}}\right|$$

$$= \left|\frac{-16 - 36 - 64}{\sqrt{116}}\right|$$

$$= \left|\frac{-116}{\sqrt{116}}\right| = \sqrt{116}$$

$= 2\sqrt{29}$ units

30. $(3 + 2x)^{50} = 3^{50}\left(1 + \frac{2}{3}x\right)^{50}$

Here, $T_{r+1} = 30^{50}\,{}^{50}C_r\left(\frac{2x}{3}\right)^r$

and $\quad T_r = 30^{50}\,{}^{50}C_{r-1}\left(\frac{2x}{3}\right)^{r-1}$

But $\quad x = \frac{1}{5} \quad$ (given)

$\therefore \frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}} \cdot \frac{2}{3} \cdot \frac{1}{5} \geq 1$

$\Rightarrow \quad 102 - 2r \geq 15r$

$\Rightarrow \quad r \leq 6$

Hence, the largest term is 7th.

31. Given, $x = a\cos^3 t$, $y = a\sin^3 t$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt} = a3\cos^2 t \cdot (-\sin t) = -3a\sin t \cdot \cos^2 t$$

and $\quad \frac{dy}{dt} = a3\sin^2 t \cdot (\cos t) = 3a\sin^2 t \cdot \cos t$

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3a\sin^2 t \cdot \cos t}{-3a\sin t \cdot \cos^2 t}$

$= -\tan t$

At $\quad t = \frac{\pi}{4}, \left(\frac{dy}{dx}\right)_{t=\pi/4} = -\tan\frac{\pi}{4} = -1$

32. Given, $f(x) = 4x^3 - 6x^2 + 2x + 5$, $F(0) = 5$

Anti-derivative of $f = F$

$\Rightarrow \quad F' = f$

$\Rightarrow \quad F = \int f\,dx + C$

$= \int (4x^3 - 6x^2 + 2x + 5)\,dx + C$

$= \left[\frac{4x^4}{4} - \frac{6x^3}{3} + \frac{2x^2}{2} + 5x\right] + C$

$\Rightarrow \quad f = x^4 - 2x^3 + x^2 + 5x + C$ ...(i)

On putting $x = 0$, we get, $F(0) = 5$

$\Rightarrow \quad 0 - 2(0) + 0 + 5(0) + C = 5$

$\Rightarrow \quad C = 5$

On putting the value of C in Eq. (i), we get

$F = x^4 - 2x^3 + x^2 + 5x + 5$

33. An intput device.

34. Let $x = \sqrt{12} \Rightarrow x^2 = 12$

$\Rightarrow \quad x^2 - 12 = 0$

$\Rightarrow \quad f(x) = x^2 - 12$

On differentiating w.r.t., x, we get

$f'(x) = 2x$

$\because \quad f(3) < 0$ and $f(4) > 0$

Hence, root will lie between 3 and 4. $|f(3)| < |f(4)|$

$\therefore \quad x_0 = 3$

First iteration,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{(9-12)}{2 \times 3} = 3 + \frac{3}{6} = 3.5$$

Now, second iteration,

$$x_2 = 3.5 - \frac{f(3.5)}{f'(3.5)}$$

$$= 3.5 - \frac{[(3.5)^2 - 12]}{2 \times 3.5}$$

$$= 3.463$$

35. Given, equation is

$x^2 - 18x + 81 = 0$

$\Rightarrow \quad x^2 - 9x - 9x + 81 = 0$

$\Rightarrow \quad x(x-9) - 9(x-9) = 0$

$\Rightarrow \quad (x-9)(x-9) = 0$

$\Rightarrow \quad x = 9, 9$

Hence, the roots of the quadratic equation are 9 and 9.

$\therefore$ Their geometric mean $= \sqrt{9 \times 9} = 9$

36. Let $f(x) = x(-1)^{[1/x]}$

Here, $\left[\frac{1}{x}\right]$ is a greatest integer function which gives either positive integer or negative integer.

Then, $f(x) = x(-1)^{\text{odd integer}}$

or $x(-1)^{\text{even integer}}$

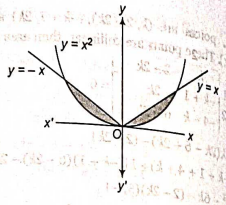
$\therefore \lim_{x \to 0} x(-1)^{[1/x]} = \lim_{x \to 0} x(-1)^{\text{odd integer}}$

or $\lim_{x \to 0} x(-1)^{\text{even integer}}$

$= 0$

37. Given, curves are $y = x^2$

and $y = |x|$



So, the intersection points of given curves are (1, 1) and (−1, 1).

$\therefore$ Required area

$= 2$ (Shaded area in first quadrant)

$$= 2\int_0^1 (x - x^2)dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 2 \times \frac{1}{6} = \frac{1}{3}$$

38. Given, centre of the circle lies on x-axis.

$\therefore$ Centre of the circle $= (-g, 0)$.

Then, equation of the circle is

$(x+g)^2 + y^2 = (\sqrt{g^2 - c})^2$

$\Rightarrow \quad x^2 + g^2 + 2xg + y^2 = g^2 - c$

$\Rightarrow \quad x^2 + 2xg + y^2 + c = 0$ ...(i)

This circle passes through the point (0, 2).

$\therefore \quad 0 + 2(0)g + (2)^2 + c = 0$

$\Rightarrow \quad c = -4$

On putting the value of c in Eq. (i), we get

$x^2 + 2xg + y^2 - 4 = 0$ ...(ii)

This circle also passes through the point (3, 3).

$\therefore \quad (3)^2 + 2(3)g + (3)^2 - 4 = 0$

$\Rightarrow \quad 6g = -14$

$\Rightarrow \quad g = -\frac{14}{6} = \frac{-7}{3}$

On putting the value of g in Eq. (ii), we get

$x^2 + 2\left(-\frac{7}{3}\right)x + y^2 - 4 = 0$

$\Rightarrow \quad 3x^2 + 3y^2 - 14x - 17 = 0$

which is required equation of circle.

39. Given, $\sin\theta = \frac{1}{2} = \sin 150°$

(since, θ is obtuse angle)

$\Rightarrow \quad \theta = 150°$

Now, $\cot\theta = \cot 150° = -\sqrt{3}$

40. Given, lines are

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$$

and $$\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2}$$

Now, $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$= (3)(1) + (-2)\left(\frac{3}{2}\right) + (0)(2)$

$= 3 - 3 + 0$

$\Rightarrow \quad \cos\theta = 0 = \cos\frac{\pi}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{2}$

41. Main memory.

42. Given, total cost, $C(x) = 3x^3 - 2x^2 + x + 100$

Marginal cost

$$= \frac{dC(x)}{dx}$$

$$= \frac{d}{dx}(3x^3 - 2x^2 + x + 100)$$

$$= 9x^2 - 4x + 1$$

When $x = 5$, then marginal cost

$= 9(5)^2 - 4 \times 5 + 1$

$= 225 - 20 + 1 = 206$

43. Given, events A and B are mutually exclusive events.

$\therefore \quad P(A \cap B) = 0$

Then, $\quad P\left(\frac{A}{B}\right) = 0$

44. Here, $a' = \frac{b \times c}{[a\,b\,c]}$, $b' = \frac{c \times a}{[a\,b\,c]}$, $c' = \frac{a \times b}{[a\,b\,c]}$

$\therefore \quad a \times a' = \frac{a \times (b \times c)}{[a\,b\,c]}$,

$b \times b' = \frac{b \times (c \times a)}{[a\,b\,c]}$

and $\quad c \times c' = \frac{c \times (a \times b)}{[a\,b\,c]}$

Now, $a \times a' + b \times b' + c \times c'$

$$= \frac{a \times (b \times c)}{[a\,b\,c]} + \frac{b \times (c \times a)}{[a\,b\,c]} + \frac{c \times (a \times b)}{[a\,b\,c]}$$

$$= \frac{1}{[a\,b\,c]}[(a \cdot c)b - (a \cdot b)c + (b \cdot a)c - (b \cdot c)a + (c \cdot b)a - (c \cdot a)b]$$

$= 0$

45. Here, $a = 0$, $b = 4$ and $n = 4$

$$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
$y = e^x$	1 (y_0)	2.72 (y_1)	7.39 (y_2)	20.09 (y_3)	54.60 (y_4)

Hence,

$$\int_0^4 e^x\,dx = \frac{h}{3}[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2]$$

$$= \frac{1}{3}(1 + 54.60) + 4(2.72 + 20.09) + 2(7.39)]$$

$$= \frac{1}{3}[55.60 + 91.24 + 14.78]$$

$$= \frac{1}{3}[161.62]$$

$$= 53.87$$

46. Given, quadratic equation is

$2x^2 + 3x + 1 = 0$

$\Rightarrow \quad 2x^2 + 2x + x + 1 = 0$

$\Rightarrow \quad 2x(x+1) + 1(x+1) = 0$

$\Rightarrow \quad (2x+1)(x+1) = 0$

$\Rightarrow \quad x = -\frac{1}{2}, -1$

Hence, the roots of the quadratic equation are rational.

47. Given, $P(A \cap B) = \frac{1}{6}$

$\Rightarrow \quad P(A) \cdot P(B) = \frac{1}{6}$ ...(i)

($\because$ A and B are independent events)

and $\quad P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

$\Rightarrow \quad P(\overline{A \cup B}) = \frac{1}{3}$

$\Rightarrow \quad 1 - P(A \cup B) = \frac{1}{3}$

$\Rightarrow \quad P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = \frac{2}{3}$

$\Rightarrow \quad P(A) + P(B) - P(A) \cdot P(B) = \frac{2}{3}$

$\Rightarrow \quad P(A) + P(B) = \frac{2}{3} + \frac{1}{6} = \frac{4+1}{6} = \frac{5}{6}$ ...(ii)

Now, $P(A) - P(B)$

$$= \sqrt{\{(P(A) + P(B))\}^2 - 4P(A) \cdot P(B)}$$

$$= \sqrt{\left(\frac{5}{6}\right)^2 - 4\left(\frac{1}{6}\right)} \quad \text{[from Eqs. (i) and (ii)]}$$

$$= \sqrt{\frac{25}{36} - \frac{4}{6}} = \sqrt{\frac{1}{36}} = \frac{1}{6}$$

$\Rightarrow \quad P(A) - P(B) = \frac{1}{6}$ ...(iii)

On adding Eqs. (ii) and (iii), we get

$$2P(A) = \frac{5}{6} + \frac{1}{6} = 1$$

$$\Rightarrow \quad P(A) = \frac{1}{2}$$

On putting the value of $P(A)$ in Eq. (iii), we get

$$\frac{1}{2} - P(B) = \frac{1}{6}$$

$$\Rightarrow \quad P(B) = \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6}$$

$$\Rightarrow \quad P(B) = \frac{1}{3}$$

Hence, $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$

48. Given, radius = a

The equation of circle passes through origin is

$$(x-h)^2 + (y-k)^2 = h^2 + k^2$$

$$(x-a)^2 + y^2 = a^2$$

Whose radias is 'a' and center lie on x-axis.

49. A system software.

50. Let $I = \int_{-1}^{2} |x^3 - x|\, dx$

$$= \int_{-1}^{0} (x^3 - x)dx + \int_{0}^{1} (-x(x^2-1))\,dx + \int_{1}^{2} (x^3 - x)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{-x^4}{4} + \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$$= \left(0 - \frac{1}{4} + \frac{1}{2}\right) + \left(-\frac{1}{4} + \frac{1}{2}\right) + \left(\frac{16}{4} - \frac{4}{2} - \frac{1}{4} + \frac{1}{2}\right)$$

$$= -\frac{1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{2} + 4 - 2 + \frac{1}{4}$$

$$= \frac{11}{4}$$