Given, radius = a

The equation of circle passes through origin is

$$(x-h)^2+(y-k)^2=h^2+k^2$$

$$(x-a)^2+y^2=a^2$$

Whose radias is 'a' and center lie on x-axis.

A system software.

Let $I=\int_{-1}^{2}|x^3-x|\,dx$

$$=\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}\{-x(x^2-1)\}dx+\int_{1}^{2}(x^3-x)dx$$

$$=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{-x^4}{4}+\frac{x^2}{2}\right]_{0}^{1}+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{1}^{2}$$

$$=\left(0-\frac{1}{4}+\frac{1}{2}\right)+\left(-\frac{1}{4}+\frac{1}{2}\right)+\left(\frac{16}{4}-\frac{4}{2}-\frac{1}{4}+\frac{1}{2}\right)$$

$$=-\frac{1}{4}+\frac{1}{2}-\frac{1}{4}+\frac{1}{2}+4-2+\frac{1}{4}$$

$$=\frac{11}{4}$$

CG PET

Engineering Entrance Exam

Solved Paper 2012

Physics

1. When an impurity is doped into an intrinsic semiconductor, the conductivity of the semiconductor
   (a) increases (b) decreases
   (c) remains the same (d) becomes zero
2. The focal length of a normal eye lens is about
   (a) 1 mm (b) 2 cm
   (c) 25 cm (d) 1 m
3. The laws of Newton's are applicable
   (a) in rotatory frame
   (b) in inertial frame
   (c) in non-inertial frame
   (d) in accelerated frame
4. When a charged particle enters normally in a magnetic field, the force acting on the particle makes it to move along a path which is a
   (a) helix (b) straight line
   (c) circle (d) None of these
5. When a mirror is rotated by an angle 30°, then the reflected ray will be rotated by an angle
   (a) 30° (b) 60°
   (c) 90° (d) 15°
6. To increase the angular magnification of a simple microscope one should increase
   (a) the focal length of lens
   (b) the power of lens
   (c) the aperture of lens
   (d) the object size
7. A point particle of mass 1.0 g is rotating in a circle of diameter 8 cm. Its moment of inertia will be
   (a) 2 g× cm^2 (b) 4 g× cm^2
   (c) 8 g× cm^2 (d) 16 g× cm^2
8. Two infinite plane parallel sheets separated by a distance d have equal and opposite uniform charge densities σ. Electric field at a point between the sheets is
   (a) zero
   (b) σ/ε_0
   (c) $\sigma/2\varepsilon_0$
   (d) depends upon the location of the point
9. X-rays beam can be deflected
   (a) by an electric field
   (b) by magnetic field
   (c) when both electric and magnetic field are present
   (d) neither by an electric field nor by magnetic field
10. Magnetic meridian is
    (a) a point
    (b) a line along north : south
    (c) a horizontal plane
    (d) a vertical plane
11. Which is the best good conductor of heat?
    (a) Iron (b) Copper
    (c) Wood (d) Aluminium
12. Gravitational potential energy is
    (a) always positive
    (b) always negative
    (c) either negative or positive
    (d) always zero
13. Which of the following expressions does not represent simple harmonic motion?
    (a) $A\cos\omega t$
    (b) $A\sin 2\omega t$
    (c) $A\sin\omega t+B\cos\omega t$
    (d) $A\sin^2\omega t$

14. If yellow light in the Young's double slit experiment is replaced by red light, the fringe width will
(a) decrease
(b) remains unaffected
(c) increase
(d) first increase then decrease

15. Calculate the resistance of an aluminium wire of length 50 cm and cross-sectional area 2.0 mm^2. The resistivity of aluminium is $e = 2.6 \times 10^{-8}\ \Omega$-m
(a) 0.0065 Ω (b) 0.065 Ω
(c) 0.1265 Ω (d) None of these

16. When nuclear fission process occurs in uranium, the major elements liberated are
(a) gold, krypton
(b) barium, sodium
(c) barium, krypton
(d) None of the above

17. A charged particle of charge q of mass m moves in a circular path of radius r in a magnetic field of intensity B. The frequency of revolution will be
(a) $Bq/2\pi m$
(b) $Bq/2\pi rm$
(c) $Bm/2\pi q$
(d) None of the above

18. In a surface tension experiment with a capillary tube water rises up to 0.1 m. If the same experiment is repeated an artificial satellite which is revolving around the earth, water will rise in the capillary tube up to a height of
(a) 0.1 m (b) 0.2 m
(c) 0.98 m (d) None of these

19. The equation of simple harmonic oscillation is $y = 5 \sin \pi (t + 4)$ m then the value of amplitude A and time period T are
(a) $A = 5, T = 2$ (b) $A = 10, T = 1$
(c) $A = 5, T = 1$ (d) None of these

20. The time constant of L-R circuit is
(a) L/R (b) R/L
(c) $1/RL$ (d) RL

21. A body 500 g is thrown vertically up with a velocity of 100 m/s. The work done by force of gravity when the body goes up is
(a) 25 J (b) 5 J
(c) –5 J (d) –25 J

22. The mass number of a nucleus is equal to
(a) the number of neutrons in the nucleus
(b) the number of protons in the nucleus
(c) the number of nucleons in the nucleus
(d) None of the above

23. Newton's law of cooling is a special case of
(a) Wien's displacement law
(b) Kirchhoff's law
(c) Stefan law
(d) Planck's law

24. The period of oscillation of a magnet in vibration magnetometer is 2 s. The period of oscillation of a magnet whose magnetic moment is four times that of first magnet is
(a) 1 s (b) 4 s (c) 8 s (d) 0.5 s

25. When the temperature of a metallic resistor is increased, the product of its resistivity and conductivity
(a) increases
(b) decreases
(c) remains constant
(d) may increase or decrease

26. Why the calorimeter is made by copper?
(a) Specific heat of copper is zero
(b) Specific heat of copper is less
(c) Specific heat of copper is higher
(d) None of the above

27. At mean point of a body executing simple harmonic motion the velocity will be
(a) zero
(b) minimum but not zero
(c) maximum
(d) infinite

28. The energy of a hydrogen atom in the ground state is
(a) –12.8 eV (b) –13.6 eV
(c) –16.5 eV (d) –15.8 eV

29. The dimension of pressure is
(a) $[ML^2T^{-2}]$ (b) $[MLT^{-2}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-2}]$

30. Critical angle for glass whose refractive index is 1.5, is
(a) 45° (b) 48° (c) 42° (d) 30°

31. Q charge is given to two capacitors C_1 and C_2 which are in parallel. The charge distribution among them is
(a) $C_1 : C_2$ (b) $C_2 : C_1$
(c) $C_1C_2 : 1$ (d) $1 : C_1C_2$

32. For a given material, the value of γ is 2.4 times that of η. Its Poissons ratio is
(a) 2.4 (b) 1.2
(c) 0.4 (d) 0.2

33. A particle moves with a constant velocity parallel to x-axis, its angular velocity with respect to origin
(a) is zero (b) goes on increasing
(c) goes on decreasing (d) remain constant

34. If the length of wire of a potentiometer is increased, its balance point will be
(a) increased (b) decreased
(c) becomes four times (d) unchanged

35. A planet has twice the values of mass and radius than that of the earth. Acceleration due to gravity on the surface of planet is
(a) 9.8 m/s^2 (b) 4.9 m/s^2
(c) 980 m/s^2 (d) 19.6 m/s^2

36. The force on a charge q placed in a uniform electric field E will be
(a) $\frac{E}{q}$ (b) qE
(c) E (d) $\frac{q}{E}$

37. Two water droplets merge with each other to form a larger droplet. In this process,
(a) energy is liberated
(b) energy is absorbed
(c) energy is neither liberated nor absorbed
(d) some mass is converted into energy

38. Escape velocity of a body on the surface of earth is 11.2 km/s. If the mass of the earth increases to twice its present value and radius of the earth becomes half, the escape velocity becomes
(a) 22.4 km/s (b) 11.2 km/s
(c) 5.6 km/s (d) None of these

39. In a isothermal process, specific heat of gas is
(a) zero (b) negative
(c) infinity (d) None of these

40. If the length of a wire is doubled its resistance becomes
(a) 2 times (b) 4 times
(c) 3 times (d) unchanged

41. If one g of substance is converted to energy, then the energy liberated is
(a) 9×10^{13} J (b) 9×10^{11} J
(c) 9×10^{16} J (d) None of these

42. A metallic hollow sphere of radius 3 cm is charged so that potential at its surface is 5 V. The potential at the centre of sphere will be
(a) 0 V (b) 5 V
(c) 3 V (d) 10 V

43. The dielectric constant of air is
(a) $8.9 \times 10^{-12}\ C^2N^{-1}m^{-2}$
(b) 1
(c) infinite
(d) None of these

44. In an adiabatic process on a gas with $\gamma = 1.4$, the pressure is increased by 0.5%. The volume will decrease
(a) 0.36% (b) 0.5%
(c) 0.7% (d) 1%

45. If a resistance $R = 10\Omega$, inductance $L = 2$ mH and capacitance $C = 5\mu F$ is connected in series to an AC source of 20 Hz. The impedance of circuit is
(a) zero (b) 10 Ω
(c) 100 Ω (d) 1000 Ω

46. Velocity of light is maximum in
(a) water (b) air
(c) glass (d) diamond

47. For total internal reflection
(a) Incident angle = Critical angle
(b) Incident angle > Critical angle
(c) Incident angle < Critical angle
(d) None of the above

48. A linear conductor of length 40 cm and carrying current 3 A is placed in a uniform magnetic field of intensity 500 Gauss. If the conductor makes an angle 30° with the direction of magnetic field, the force acting on it as
(a) 3×10^4 N (b) 3×10^2 N
(c) 3×10^{-2} N (d) None of these

49. Cathode rays are similar to
(a) X-rays (b) γ-rays
(c) α-particles (d) β-particles

50. A p-type semiconductor is
(a) positively charged
(b) negatively charged
(c) uncharged
(d) uncharged at 0 K but charged at high temperatures

Chemistry

1. NO_3^- is detected by ring test. Ring formed has formula
(a) $FeSO_4 \cdot HNO_2$ (b) $FeSO_4 \cdot NO_2$
(c) $[Fe(H_2O)_5 \cdot NO]^{2+}$ (d) $Fe(OH)_2 \cdot NO$

2. A radioactive substance disintegrate its 90% in 100 days. How much time it will take to disintegrate its 99.9%?
(a) 100 days (b) 200 days
(c) 300 days (d) 400 days

3. Nylon-6 is made from
(a) 1, 3-butadiene (b) chloroprene
(c) adipic acid (d) caprolactum

4. Formula of hexaaqua manganese (II) phosphate is
(a) $[Mn(H_2O)]_6PO_4$
(b) $[Mn(H_2O)_6]_3PO_4$
(c) $[Mn(H_2O)_6]_3(PO_4)_2$
(d) $[Mn(H_2O)_6](PO_4)_3$

5. Which transition element of the following outermost electronic configuration shows the highest oxidation state?
(a) $3d^3, 4s^2$ (b) $3d^5, 4s^1$
(c) $3d^5, 4s^2$ (d) $3d^6, 4s^2$

6. Which of the following has magnesium?
(a) Haemoglobin (b) Chlorophyll
(c) Vitamin B_{12} (d) Haemocyanin

7. Bakelite is the condensation polymer of
(a) C_6H_5OH and caprolactum
(b) HCHO and phthalic acid
(c) C_6H_5OH and HCHO
(d) HCHO and ethylene glycol

8. Which of the following is not a method of preparation of colloidal solution?
(a) Electrical dispersion
(b) Peptization
(c) Coagulation
(d) Mechanical dispersion

9. The process of oxidation involve
(a) loss of electron (b) gain of electron
(c) loss of proton (d) loss of neutron

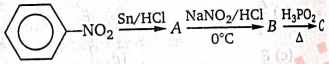
10. Fibrous protein are not present in
(a) myosin (b) albumins
(c) collagen (d) fibroin

11. The formula of chromyl chloride is
(a) CrO_2Cl_2 (b) $CrOCl_2$
(c) $CrCl_3$ (d) Cr_2OCl_2

12. Pure nitrogen gas is obtained from
(a) $NH_3 + NaNO_2$ (b) $NH_4Cl + NaNO_2$
(c) $N_2O + Cu$ (d) $(NH_4)_2Cr_2O_7$

13. The Modern Periodic Table is based on the properties of elements as a function of
(a) atomic number (b) atomic mass
(c) electronegativity (d) electron affinity

14. The pH of 0.05 M solution of a strong dibasic acid is
(a) 0.0 (b) 1.0
(c) 0.2 (d) 0.5

15. 

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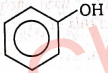
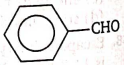
(a) 

(b) 
(c) 

(d) 

16. What is the magnetic moment value of Fe^{2+} ion in $[Fe(CN)_6]^{4-}$?
(a) 0 (b) $\sqrt{3}$
(c) $\sqrt{24}$ (d) $\sqrt{8}$

17. In the reaction,

$$CH_3CHO \xrightarrow{[O]} A \xrightarrow{P, Cl_2} B \xrightarrow{KCN} C \xrightarrow{H^+, H_2O} D$$

The end product D is
(a) crotonic acid
(b) lactic acid
(c) malonic acid
(d) tartaric acid

18. The molarity of pure water is
(a) 18 (b) 5.56
(c) 55.6 (d) 100

19. Inorganic preservative is
(a) benzoic acid
(b) sucrose
(c) vinegar
(d) potassium bisulphite

20. The attacking agent in the electrophilic sulphonation of benzene is
(a) SO_4^{2-} (b) SO_3^{2-}
(c) SO_2 (d) SO_3

21. The IUPAC name of the following compound
$CH_3—CH=CH—C\equiv CH$ is
(a) pent-2-en-4-yne (b) pent-1-en-4-yne
(c) pent-3-en-1-yne (d) pent-2-en-5-yne

22. Which of the following represents the first law of thermodynamics?
(a) $\Delta U = q + W$ (b) $\Delta U = p \cdot \Delta V$
(c) $\Delta U = \Delta H + P \cdot \Delta V$ (d) $\Delta H = q + W$

23. The standard reduction potential $E°$ for half-reactions are

$$Zn \longrightarrow Zn^{2+} + 2e^- ; E° = +0.76 \text{ V}$$
$$Fe \longrightarrow Fe^{2+} + 2e^- ; E° = +0.41 \text{ V}$$

The EMF of the cell reaction
$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$ is
(a) + 1.28 V (b) –1.28 V
(c) + 0.35 V (d) –0.35 V

24. Aqueous solution of $Na_2S_2O_3$ on reaction with Cl_2 gives
(a) $Na_2S_4O_6$ (b) NaOH
(c) $NaHSO_4$ (d) NaCl

25. What number of subshells are possible in $n = 3$ energy level
(a) 7 (b) 8
(c) 9 (d) 6

26. 2-acetoxy benzoic acid is
(a) antiseptic (b) analgesic
(c) antibiotic (d) hypnotic

27. The lanthanoid contraction is responsible for the fact that
(a) Zn and Y have about same radius
(b) Zr and Nb have similar oxidation states
(c) Zr and Hf have about the same radius
(d) Zr and Zn have the same oxidation state

28. Which of the following is not present in DNA?
(a) Adenine (b) Guanine
(c) Uracil (d) Thymine

29. $C_2H_2 \xrightarrow[H_2SO_4, H_2O]{Hg^{2+}} A \xrightarrow{[O]} B$

The compound B is
(a) an acid (b) an aldehyde
(c) ketone (d) ethanol

30. The number of electrons transferred when $KMnO_4$ acts as an oxidising agent to give MnO_2 and Mn^{2+} respectively are
(a) 2, 3 (b) 1, 5
(c) 3, 5 (d) 1, 3

31. For the equilibrium reaction,

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) + \Delta H$$

the increase in temperature
(a) favours the formation of SO_3
(b) favours the decomposition of SO_3
(c) does not affect the equilibrium
(d) stops the reaction

32. The property of crystalline solid is not
(a) anisotropic (b) isotropic
(c) hardness (d) denseness

33. What is the electronic configuration of O^{2-} ion?
(a) $1s^2, 2s^2, 2p^5, 3s^1$ (b) $1s^2, 2s^2, 2p^4, 3s^2$
(c) $1s^2, 2s^2, 2p^6$ (d) $1s^2, 2s^2, 2p^4$

34. Hinsberg's reagent is
(a) benzene sulphonyl chloride
(b) benzene sulphonic acid
(c) phenyl isocyanide
(d) benzene sulphonamide

35. Froth floatation process is used for the concentration of which one?
(a) Horn silver (b) Haematite
(c) Bauxite (d) Cinnabar

36. An organic compound containing C, H and N gives the following on analysis : C = 40%, H = 13.33% and N = 46.67%. What would be its empirical formula?
(a) C_2H_7N (b) $C_2H_7N_2$
(c) CH_4N (d) CH_3N

37. The normality of 0.2 M H_3PO_2 is
(a) 0.2 N (b) 0.4 N
(c) 0.6 N (d) 0.06 N

38. Benzene has the following hybridisation
(a) sp (b) sp^2
(c) sp^3 (d) dsp^2

39. Propyne and propene can be distinguished by
(a) conc. H_2SO_4
(b) Br_2 in CCl_4
(c) dil. H_2SO_4
(d) $AgNO_3$ in ammonia

40. A common metal used for the extraction of metals from their oxides by reduction is
(a) Al (b) Fe
(c) Cr (d) Co

41. Which of the following does not give silver mirror test?
(a) HCOOH (b) CH_3CH_2CHO
(c) CH_3CHO (d) CH_3COCH_3

42. Due to Frenkel defect, the density of the ionic solids
(a) does not change
(b) decreases
(c) increases
(d) may increase or decrease

43. The colour of the flame of sodium is
(a) deep red (b) blue
(c) green (d) golden yellow

44. Which of the following is the strongest reducing agent?
(a) HF (b) HCl
(c) HI (d) HBr

45. Cetyltrimethyl ammonium bromide (TAB) is popular as
(a) antioxidant (b) antibiotic
(c) surfactant (d) tranquillizer

46. A first order reaction completed its 20% in 200 minute. How much time it will take to complete its 80%?
(a) 400 min (b) 800 min
(c) 1400 min (d) 1000 min

47. Which of the following metal is leached by cyanide process?
(a) Na (b) Cu (c) Ag (d) Fe

48. The half-life period ($t_{1/2}$) for ^{241}Am in years, (given that it emits 1.2×10^{11} α-particles per gram per second) is
(a) 375 yr (b) 112 yr
(c) 615 yr (d) 458 yr

49. If the initial concentration of the reactant is doubled, the time for half reaction is also doubled. The order of the reaction is
(a) zero (b) one
(c) two (d) three

50. Heat of neutralization of HCl and NaOH is
(a) –13.7 kcal (b) –13.7 J
(c) –13.7 kJ (d) None of these

Mathematics

1. If θ is the angle between vectors $\mathbf{p}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ and $\mathbf{q}=b\mathbf{i}+c\mathbf{j}+a\mathbf{k}$, then θ lies in
(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[\frac{\pi}{2}, \pi\right]$
(c) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ (d) $\left[0, \frac{2\pi}{3}\right]$

2. If the integer λ and μ are chosen at random between 1 and 100, then the probability that a number of the form $7^\lambda + 7^\mu$ is divisible by 5 is
(a) 1/4 (b) 1/7
(c) 1/8 (d) 1/49

3. If x is a complex root of the equation $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0$, then $x^{2005} + \frac{1}{x^{2005}}$ is equal to
(a) 1 (b) –1 (c) i (d) ω

4. For $f(x) = (x-1)^{2/3}$, the mean value theorem is applicable to $f(x)$ in the interval
(a) [2, 4] (b) [0, 2]
(c) [–2, 2] (d) any finite interval

5. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is
(a) 0 (b) –1
(c) 1 (d) 2

6. If a line makes angles α, β, γ and δ with the four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is
(a) 1 (b) 2 (c) 2/3 (d) 4/3

7. The Newton Raphson method converges fast, if $f'(\alpha)$ is
(a) small (b) large
(c) 0 (d) None of these

8. If n is positive integer, then $(1+i)^n + (1-i)^n$ is equal to
(a) $(\sqrt{2})^{n-2}\cos\frac{n\pi}{4}$ (b) $(\sqrt{2})^{n-2}\sin\frac{n\pi}{4}$
(c) $(\sqrt{2})^{n+2}\cos\frac{n\pi}{4}$ (d) $(\sqrt{2})^{n+2}\sin\frac{n\pi}{4}$

9. The value of $\int_0^1 \sin^{-1} x\, dx$ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} - 1$
(c) $\frac{\pi}{2} + 1$ (d) 0

10. If $\lim_{x\to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$, then
(a) $a = -\frac{5}{2}, b = -\frac{1}{2}$ (b) $a = -\frac{3}{2}, b = -\frac{1}{2}$
(c) $a = -\frac{3}{2}, b = -\frac{5}{2}$ (d) $a = -\frac{5}{2}, b = -\frac{3}{2}$

11. The solution of the differential equation $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is
(a) $\sin\frac{x}{y} = x + C$ (b) $\sin\frac{y}{x} = Cx$
(c) $\sin\frac{x}{y} = Cy$ (d) $\sin\frac{y}{x} = Cy$

12. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then x is equal to
(a) $\log_b(\log_a b)$
(b) $\log\log(a)$
(c) $\log(\log a) + \log(\log b)$
(d) $\log_a(\log_b a)$

13. At the foot of a mountain the elevation of its summit is 45°. After ascending 2 km towards the mountain up an incline of 30°, the elevation changes to 60°. The height of mountain will be
(a) $\sqrt{3} - 1$ (b) $1 - \sqrt{3}$
(c) $1 + \sqrt{3}$ (d) $\sqrt{3}$

14. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-coplanar vectors and $\mathbf{p} = \frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, $\mathbf{q} = \frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$ and $\mathbf{r} = \frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$, then $\mathbf{a}\cdot\mathbf{p} + \mathbf{b}\cdot\mathbf{q} + \mathbf{c}\cdot\mathbf{r}$ is equal to
(a) 3
(b) –3
(c) 0
(d) None of the above

15. The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is
(a) 45° (b) 30°
(c) 60° (d) 90°

16. The value of $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$ is
(a) $\cos^{-1}\frac{36}{85}$ (b) $\sin^{-1}\frac{15}{85}$
(c) $\sin^{-1}\frac{36}{85}$ (d) $\cos^{-1}\frac{15}{85}$

17. The ratio in which the join of (1, –2, 3) and (4, 2, –1) is divided by the XOY plane is
(a) 1 : 3 (b) 3 : 1
(c) –1 : 3 (d) None of these

18. The arithmetic calculations in a computer are performed by
(a) input unit (b) ALU
(c) output unit (d) memory

19. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is
(a) $\frac{1}{x}$ (b) x
(c) e^{-x} (d) e^{-y}

20. If $\frac{1+4p}{4}, \frac{1-p}{4}, \frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then
(a) $\frac{1}{3} \le p \le \frac{1}{2}$ (b) $\frac{1}{3} \le p \le \frac{2}{3}$
(c) $\frac{1}{6} \le p \le \frac{1}{2}$ (d) None of these

21. If $\frac{a+b}{1-ab}, \frac{b+c}{1-bc}$ are in AP, then $a, \frac{1}{b}$ and c are in
(a) AP (b) GP
(c) HP (d) None of these

22. If A and B are two events, then $P(\bar{A}\cap B)$ is equal to
(a) $P(\bar{A})P(\bar{B})$
(b) $1 - P(A) - P(B)$
(c) $P(A) + P(B) - P(A\cap B)$
(d) $P(B) - P(A\cap B)$

23. If $m = {}^nC_2$, then ${}^mC_2$ is equal to
(a) $n + {}^1C_4$ (b) $3\times{}^nC_4$
(c) $3\times{}^{n+1}C_4$ (d) None of these

24. Let $f(x) = a - (x-3)^{8/9}$, then maxima of $f(x)$ is
(a) 3 (b) $a - 3$
(c) a (d) None of these

25. By Simposon's rule taking $n = 4$, the value of the integral $\int_0^1 \frac{dx}{1+x^2}$ is equal to
(a) 0.785 (b) 0.788
(c) 0.781 (d) None of these

26. The differential coefficient of $f(\log x)$ with respect to x, where $f(x) = \log x$ is
(a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$
(c) $(x \log x)^{-1}$ (d) None of these

27. The control structure of IF-THEN is a
(a) multiple selection (b) double selection
(c) single selection (d) None of these

28. If $b + c = 3a$, then the value of $\cot \frac{B}{2} \cot \frac{C}{2}$ is
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 2 (d) 1

29. The centre of the circle which circumscribes the square formed by $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is
(a) (4, 5) (b) (3, 4)
(c) (9, 5) (d) (4, 7)

30. The value of
$\tan 57° - \tan 12° - \tan 57° \tan 12°$ is
(a) tan 69° (b) tan 45°
(c) 0 (d) None of these

31. If **a** and **b** are two vectors of magnitude 2 inclined at an angle 60°, then the angle between **a** and **a** + **b** is
(a) 30° (b) 60°
(c) 45° (d) None of these

32. The value of integral $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})\, dx$ is
(a) $\sqrt{2}\pi$ (b) π
(c) $\frac{\pi}{2}$ (d) 0

33. The value of $\int_0^1 x\, dx$ by Trapezoidal rule taking $x = 4$ is
(a) 0.34375 (b) 0.5
(c) 0.38387 (d) 0.35367

34. The area of the figure bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is
(a) 1 sq unit (b) 2 sq units
(c) 3 sq units (d) 4 sq units

35. Two dice are thrown simultaneously. The probability of getting a pair of ACE is
(a) 1/36 (b) 1/3
(c) 1/6 (d) None of these

36. If $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, then $\frac{\text{Projection of a on b}}{\text{Projection of b on a}}$ is equal to
(a) 1 (b) 7/3
(c) 3/7 (d) −1/6

37. If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then, $f(x)$ is
(a) $x^3 + \frac{1}{x^4} - \frac{129}{8}$ (b) $x^4 + \frac{1}{x^3} + \frac{129}{8}$
(c) $x^3 + \frac{1}{x^4} + \frac{129}{8}$ (d) $x^4 + \frac{1}{x^3} - \frac{129}{8}$

38. If $f(x) = \begin{cases} [x] + [-x], & x \neq 2 \\ \lambda, & x = 2 \end{cases}$, then f is continuous at $x = 2$, provided λ is equal to
(a) 1 (b) 0
(c) −1 (d) 2

39. If $A^2 - A + I = 0$, then matrix A^{-1} will be equal to
(a) $A + I$ (b) $I - A$
(c) A (d) $A - I$

40. The derivative of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ with respect to $\sqrt{1 - x^2}$ at $x = 1/2$ will be
(a) 1/4 (b) $\sec^{-1}(1/4)$
(c) 4 (d) 0

41. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$ are
(a) 1, 4 (b) 3, 4
(c) 2, 4 (d) 3, 2

42. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is equal to
(a) $\tan^2\theta$ (b) $\sec^2\theta$
(c) $\sec\theta$ (d) $|\sec\theta|$

43. The difference of the numbers $(1100110011)_2$ and $(1101001011)_2$ in binary system is
(a) $(100000)_2$ (b) $(101010)_2$
(c) $(11000)_2$ (d) $(10111)_2$

44. The value of $2, 6, 10 \ldots (4n-6)(4n-2)$ is equal to
(a) $C(2n, n)$
(b) $(n+1)(n+2)(n+3)\ldots(2n)$
(c) $n!P(2n, n)$
(d) None of above

45. If e_1 and e_2 are the eccentricities of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, then the value of $\frac{1}{e_1^2} + \frac{1}{e_2^2}$ is
(a) 3 (b) 2
(c) 1 (d) $\frac{1}{2}$

46. The solution of the differential equation $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$ is
(a) $xe^{\tan^{-1} y} = (1 - \tan^{-1} y)e^{\tan^{-1} y} + C$
(b) $xe^{\tan^{-1} y} = (\tan^{-1} y - 1)e^{\tan^{-1} y} + C$
(c) $x = \tan^{-1} y - 1 + Ce^{-\tan^{-1} y}$
(d) None of the above

47. The foot of perpendicular from the point (3, 4, 5) to the plane $x + y + z = 9$ is
(a) (2, 3, 4) (b) (3, 5, −2)
(c) (3, 5, 2) (d) (3, 2, 4)

48. The point of contact of $3x + 4y + 7 = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ is
(a) (1, 1) (b) (−1, 1)
(c) (1, −1) (d) (−1, −1)

49. The equation $x^2 - 7xy + 12y^2 = 0$ represents a
(a) circle
(b) pair of parallel straight lines
(c) pair of perpendicular straight lines
(d) pair of non-perpendicular straight lines

50. For how many values of x in the interval $[-4, -1]$ the matrix $\begin{bmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix}$ is singular?
(a) 2 (b) 1
(c) 0 (d) 3

Answers

Physics

1. (a)	2. (c)	3. (b)	4. (c)	5. (a)	6. (b)	7. (d)	8. (b)	9. (d)	10. (d)
11. (b)	12. (b)	13. (d)	14. (c)	15. (a)	16. (c)	17. (a)	18. (d)	19. (a)	20. (a)
21. (*)	22. (c)	23. (c)	24. (a)	25. (c)	26. (b)	27. (c)	28. (b)	29. (c)	30. (c)
31. (a)	32. (b)	33. (c)	34. (b)	35. (b)	36. (b)	37. (a)	38. (a)	39. (c)	40. (a)
41. (a)	42. (b)	43. (b)	44. (a)	45. (*)	46. (b)	47. (b)	48. (c)	49. (d)	50. (b)

Chemistry

1. (c)	2. (c)	3. (d)	4. (c)	5. (c)	6. (b)	7. (c)	8. (c)	9. (a)	10. (b)
11. (a)	12. (b)	13. (a)	14. (b)	15. (b)	16. (c)	17. (c)	18. (c)	19. (d)	20. (d)
21. (c)	22. (a)	23. (c)	24. (c)	25. (c)	26. (b)	27. (c)	28. (c)	29. (a)	30. (c)
31. (b)	32. (b)	33. (c)	34. (a)	35. (d)	36. (c)	37. (a)	38. (b)	39. (d)	40. (a)
41. (d)	42. (a)	43. (d)	44. (c)	45. (c)	46. (c)	47. (c)	48. (d)	49. (a)	50. (a)

Mathematics

1. (d)	2. (a)	3. (a)	4. (d)	5. (b)	6. (d)	7. (b)	8. (c)	9. (b)	10. (d)
11. (b)	12. (d)	13. (c)	14. (a)	15. (d)	16. (a)	17. (b)	18. (b)	19. (a)	20. (d)
21. (c)	22. (d)	23. (c)	24. (d)	25. (a)	26. (c)	27. (a)	28. (c)	29. (d)	30. (b)
31. (a)	32. (a)	33. (b)	34. (d)	35. (a)	36. (b)	37. (d)	38. (c)	39. (b)	40. (c)
41. (d)	42. (d)	43. (c)	44. (d)	45. (c)	46. (b)	47. (a)	48. (d)	49. (d)	50. (b)

Hints & Solutions

Physics

1. When an impurity is doped into an intrinsic semiconductor, it becomes an extrinsic semiconductor in which there would be the majority of electron or the holes, due to which its conductivity increases.

2. Focal length of the lens of the normal eyes is 25 cm.

3. The laws of Newton are applicable in an intertial frame of reference. If frame is not inertial, then in order to apply the law of Newton a pseudo force is applied on the considered body.

4. When a charged particle enters normally in a magnetic, the force acting on the particle makes it to move along a path which is a circle.

5. When a mirror is rotated by an angle 30°, the reflected ray will also be rotated by an angle 30°, became in each and every condition, from the law of reflection;

   Angle of incidence = angle of reflection

   If angle of incidence is changed by 30°, then there must be change of angle of 30° to angle of reflection.

6. In simple microscope, angular magnification can be made large by choosing the focal length f small, *i.e.*, power of lens is greater.

7. As moment of inertia is given by

$$I = mr^2$$

   where, r = radius of the circle

$$= \frac{8 \text{ cm}}{2} = 4 \text{ cm}$$

$$\therefore \quad I = 1 \text{ g} \times (4 \text{ cm})^2$$

$$\Rightarrow \quad I = 16 \text{ g cm}^2$$

8. As electric field intensity between the capacitor only depends on the two quantities magnitude of surface charge density on either of the plate and permittivity of the medium between the plates and is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

   ε_0 is used for space.

9. Because X-rays is not a charged particle, so it is not deflected neither by electric field nor by the magnetic field.

10. Magnetic meridian is a plane passing through geomagnetic poles that is, through to dipole axis of the earth and a point P (on the earth's surface) is called magnetic meridian at the point P.

11. Copper is the best conductor of heat due to its better thermal conductivity (385 $\text{Wm}^{-1}\text{K}^{-1}$).

    In copper, there are a large number of free electrons which can move freely anywhere in the body of the metal. These free electrons helps in carrying the thermal energy from one place to another in the metal.

12. Gravitational potential energy is always negative because it is always, the work done by the external agent and on the system.

13. The general equation of SHM is

$$x = A \sin \omega t$$

    Which can be (in some condition)

$$x = 2A \sin 2\omega t \text{ or } A \cos 2\omega t \text{ etc.}$$

    and the equation $A \sin \omega t + B \cos \omega t$ may be represent the resultant of two SHM

    But $A \sin^2 \omega t$ cannot show SHM.

14. As we know that fringe width, $\beta = \frac{\lambda D}{d}$, where d and D are constants.

$$\Rightarrow \quad \beta \propto \lambda$$

    So, there is a increase in β with increase in λ. Since the wavelength of red colour is greater than wavelength of yellow colour.

15. As we know that,

$$R = \rho \frac{l}{a}$$

    Given, $\rho = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$

$$a = 2\,\text{mm}^2 = 2\times10^{-6}\,\text{m}^2$$

$$\Rightarrow \quad R = 2.6\times10^{-8}\,\Omega\text{-m}\times\frac{50\times10^{-2}\,\text{m}}{2\times10^{-6}\,\text{m}^2}$$

$$\Rightarrow \quad R = 0.0065\,\Omega$$

16. A variety of combinations of the middle weight nuclei may be formed due to the fission.

For example,

$$^{236}_{92}U \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2n$$

and $$^{236}_{92}U \rightarrow {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n$$

17. The frequency of revolution (also called cyclotron frequency) is given by

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m}$$

(T = Time period)

where, $$T = \frac{2\pi r}{v_{(\text{speed})}} = \frac{2\pi m}{qB}\left(\text{As, } v = \frac{qBr}{m}\right)$$

18. As height of the liquid in the capillary tube is given by

$$h = \frac{2S\cos\theta}{\rho r g}$$

where, S = Surface tension

θ = Angle of contact

ρ = Density of the lequid

r = Radius of the capillary

g = Gravitational acceleration

As $g = 0$, h will be infinity, so option (d) is true.

19. As we know that standard equation of SHM is given by

$$y = A\sin(\omega t + \delta) \quad \text{...(i)}$$

and given equation is

$$y = 5\sin\pi(t + 4) \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$A = 5 \text{ and } T = 2$$

$$(\text{As, } \omega t = \pi t \therefore 2\pi f t = \pi t \Rightarrow f = \frac{1}{2} = \frac{1}{T} \therefore T = 2)$$

20. As we know that when an L-R circuit is connected to the battery of emf ε, the current at any time, t is given by

$$i = \frac{\varepsilon}{R}(1 - e^{-tR/L}) = \frac{\varepsilon}{R}(1 - e^{-t/T})$$

Here, T = Time constant

R = Resistance of the circuit

and L = Inductance of the inductor used in the circuit

21. As $m = 0.5$ kg, $v = 100$ m/s

$\therefore$ h = maximum height attained by the mass

$$= \frac{v^2}{2g} = \frac{100\times100}{2\times10}$$

(take $g = 10\,\text{m/s}^2$)

$$\Rightarrow \quad h = 500\text{ m}$$

Now, energy $= mgh = 0.5\times10\times500 = 2500$ J

When earth is doing a work

$\therefore$ Work $= -2500$ J

22. The mass number of a nucleus is equal to the number of nucleons in the nucleus (*i.e.*, number of protons and number of neutrons in the nucleus).

23. Newton's law of cooling is a special case of Stefan's law.

Stefan's law "The energy of thermal radiation emitted per unit time by a black body of surface area A is given by $u = \sigma A T^4$"

and from this equation Newton derived

$$\frac{dT}{dt} = -bA(T - T_0) \quad (T = \text{temperature})$$

or $$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

24. As we know that,

Time period, $$T = 2\pi\sqrt{\frac{1}{MB_H}} = 2\text{ s}$$

when $M \rightarrow 4M$

Then, $$T' = 2\pi\sqrt{\frac{I}{4MB_H}} = \frac{1}{2}2\pi\sqrt{\frac{I}{4MB_H}}$$

$$= \frac{1}{2}\times2 = 1\text{ s}$$

25. As we know that (for a material)

$$\text{Resistivity} = \frac{1}{\text{conductivity}}$$

$\therefore$ (Resistivity) (conductivity) = 1 = constant

26. Copper is used to make the calorimeter because of the specific heat capacity of copper is less.

Specific heat capacity of copper,

$$C = 0.386\left(\ln\frac{J}{g\,K}\right)$$

$$C = 0.0923\left(\ln\frac{cal}{g\,K}\right)$$

$$C = 24.5\left(\ln\frac{J}{mol\,K}\right)$$

27. At mean position velocity is maximum while the acceleration is minimum.

28. The energy of a hydrogen atom in the ground state is 13.6 eV.

29. As, pressure $= \dfrac{\text{Force}}{\text{Area}}$

$$= \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$$

30. As, we know that

$$\sin C = \frac{1}{\mu}$$

$$\therefore \quad C = \text{Critical angle} = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$= \sin^{-1}\left(\frac{1}{1.5}\right) = \sin^{-1}\left(\frac{2}{3}\right)$$

$$\therefore \quad C = 42°$$

31. As, capacitors are in parallel therefore voltage across the capacitor will be same

$$\Rightarrow \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

where, Q_1 = charge on the capacitor with capacitance C_1

Q_2 = charge on the capacitor with capacitance C_2

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

$$\therefore \quad Q_1 : Q_2 = C_1 : C_2$$

32. $$\gamma = 2.4\eta$$

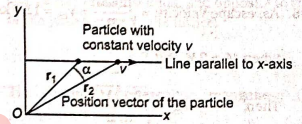
$$\eta = \frac{\gamma}{2(1 + V)}$$

$$\Rightarrow \quad \eta = \frac{2.4\eta}{2(1 + V)}$$

$$\Rightarrow \quad 2 + 2V = 2.4$$

$$\therefore \quad V = \frac{0.4}{2} = 0.2$$

33.



As, we know that

$$v = \omega r$$

$$\Rightarrow \quad \omega = \frac{v}{r}$$

$$\omega \propto \frac{1}{r} \quad (\text{As } v = \text{constant})$$

So, with the increase in position vector r will must be decreased.

34. As, we know that for potentiometer reading

$$V_0 = \varepsilon\frac{L}{l_0}$$

Here, l_0 = length of balance point and L = total length of the potentiometer wire.

As, V_0 = constant and ε = constant

The value l_0 decreased with the increase in L.

35. As, we know that the acceleration,

$$g = \frac{GM}{R^2}$$

when $M = 2M$ and $R = 2R$, then new acceleration,

$$g' = \frac{G2M}{(2R)^2} = \frac{1}{2}\frac{GM}{R^2}$$

$$\Rightarrow \quad g' = \frac{g}{2} = \frac{9.8}{2}$$

$$= 4.9\text{ m/s}^2$$

36. Electric field is defined as

'The experienced by unit charge in the field'

$$\Rightarrow \quad E = \frac{F}{q}$$

$$\therefore \quad F = qE$$

37. When two water droplets merges with each other to form a larger droplet, energy is liberated. Here the sum of the surface energies of small droplet will be greater than the surface energy of bigger one.

38. As, escape velocity $= \sqrt{\frac{2GM}{R}} = v_{es} = 11.2\,\text{km/s}$

when $M = 2M$ and $R = \frac{R}{2}$

Then, $v'_{es} = \sqrt{\frac{2G2M}{R/2}}$

$= 2\sqrt{\frac{GM}{R}}$

$= 22.4\,\text{km/s}$

39. As, we know that

$$C = \frac{\Delta Q}{n\,\Delta T}$$

where, n = number of moles of the gas and ΔT = change in temperature in isothermal process

$$\Delta T = 0$$

$\therefore C$ becomes infinity.

40. As resistance, $R = \rho\frac{l}{a}$

where, ρ = specific resistance or resistivity of the material

l = length of the wire

a = cross-sectional area of the wire

when $l = 2l$

Then, $R' = \rho\frac{2l}{a} = 2\rho\frac{l}{a}$

$\Rightarrow R' = 2R$

41. We know that,

$$E = mc^2$$

$$= 1 \times 10^{-3} \times (3 \times 10^8)^2 \text{ kg-s}$$

$$\Rightarrow E = 9 \times 10^{13} \text{ J}$$

$$= 9 \times 10^{13} \text{ J}$$

42. The potential inside the hollow sphere remains same at energy point and equal to the potential at surface of the sphere.

43. Dielectric constant for the air is 1.

44. As we know that, for adiabatic process,

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\Rightarrow \left(\frac{V_1}{V_2}\right)^\gamma = \frac{P_2}{P_1} = \frac{1.005P_1}{P_1}$$

$$\left(\because P_2 = P_1 + P_1 + \frac{5}{100} = 1.005P_1\right)$$

$$\Rightarrow \frac{V_1}{V_2} = (1.005)^{1/\gamma} = (1.005)^{1/1.4}$$

Taking $\log_{10}$ on both side

$$\log\frac{V_1}{V_2} = \frac{1}{1.4}\log 1.005$$

After process of log and antilog, we get

$$\frac{V_1}{V_2} = 1.003$$

$\therefore$ % decrease in volume = 0.36

45. For AC circuit,

$$\text{Impedance, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Here given $\omega = 2\pi f = 2\pi \times 20\text{ Hz} = 40\pi$

$R = 10\,\Omega$, $C = 5 \times 10^{-6}$ F and $L = 2 \times 10^{-3}$ H

$$\Rightarrow \frac{1}{\omega C} = 1592.11$$

and $\omega L = 0.25$

$$\therefore \left(\frac{1}{\omega C} - \omega L\right)^2 = 2539814.25$$

$$\Rightarrow Z = \sqrt{100 + 2534814.25}$$

$$= 1592.14\,\Omega$$

46. Velocity of light maximum for the air.

47. Incident angle is greater than angle because for critical angle in denser medium the angle of refraction will be a right angle further increment in angle of incidence leads the reflection of the ray in the same denser medium instead of refraction.

48. As, force on the wire

$$F = ilB\sin\theta$$

$$\Rightarrow F = 3 \times 40 \times 10^{-2} \times \frac{500}{10^4} \times \sin 30°$$

(Given, $l = 40$ cm $= 40 \times 10^{-2}$ m, $i = 3$ A and $B = 500$ gauss $= \frac{500}{10^4}$ T)

$$\therefore F = 3 \times 10^{-2}\text{ N} \quad (\sin 30° = 1/2)$$

49. Properties of β-rays, cathode rays thermions, photoelectrons, etc., are all identical except for their origin. Beta particles are created at the time of nuclear transformation, whereas in cathode rays, thermions etc., the electrons are already present and get rejected.

50. It should be remembered to the mind that the crystal (semiconductor) maintains an overall charge neutrality as the charge of additional charge carriers is just equal and opposite to that of the joined cores in the lattice.

Chemistry

1. When nitrate salt is treated with dil HCl and freshly prepared solution of $FeSO_4$ and some conc. H_2SO_4 is added by the sides of test tube, a brown ring of ferrous nitrososulphate is formed. It is called ring test of NO_3^- ion.

$$6FeSO_4 + 2HNO_3 + 3H_2SO_4 \longrightarrow 3Fe_2(SO_4)_3 + 2NO + 4H_2O$$

$$[Fe(H_2O)_6]SO_4 + NO \longrightarrow [Fe(H_2O)_5NO]SO_4 + H_2O$$

or

$$[Fe(H_2O)_5NO]^{2+}$$

2. All radioactive changes follow first order kinetics.

$\therefore$ For the 90% completion of the reaction,

$$k = \frac{2.303}{100}\log\frac{100}{100-90}$$

$$= \frac{2.303}{100}\log\frac{100}{10} = \frac{2.303}{100} \quad \text{...(i)}$$

For 99.9% completion of the reaction,

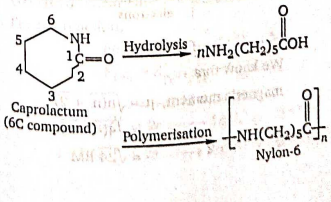
$$k = \frac{2.303}{t}\log\frac{100}{100-99.9} = \frac{2.303}{t}\log 10^3 \quad \text{...(ii)}$$

On putting the value of k from Eq. (i), we get

$$\frac{2.303}{100} = \frac{2.303}{t} \times 3\log 10$$

$$t = 3 \times 100 \text{ day} = 300 \text{ day}$$

3. Nylon-6 is obtained from caprolactum.



4. The formula of hexaaqua manganese (II) phosphate is

$$[Mn(H_2O)_6]^{2+}\ PO_4^{3-}$$

or

$$[Mn(H_2O)_6]_3(PO_4)_2$$

5. Maximum oxidation states shown by a transition element $= (n-1)d$ unpaired electrons $+ ns$ electrons.

For $3d^3, 4s^2$,

Maximum oxidation state $= 3 + 2 = 5$

For $3d^5, 4s^1$,

Maximum oxidation state $= 5 + 1 = 6$

For $3d^5, 4s^2$

Maximum oxidation state $= 5 + 2 = 7$

For $3d^6, 4s^2$,

Maximum oxidation state $= 4 + 2 = 6$

$\left[\because 3d^6 = \boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\right]$ four unpaired electrons

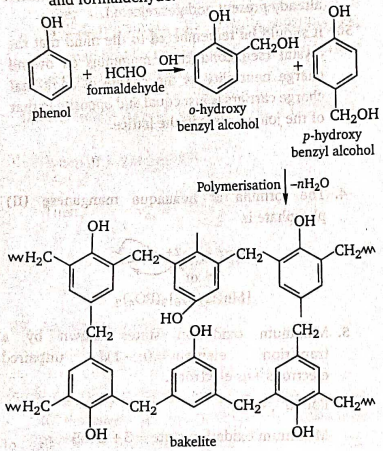
Thus, $3d^5, 4s^2$ configuration shows the highest oxidation state.

6.

Biomolecule	Metal present
Haemoglobin	Iron (Fe)
Chlorophyll	Magnesium (Mg)
Vitamin-B_{12}	Cobalt (Co)
Haemocyanin	Copper (Cu)

Thus, chlorophyll contains Mg.

7. Bakelite is a condensation copolymer of phenol and formaldehyde.



8. Coagulation is the process of precipitation or settling of colloidal particles. It is not a method of their preparation.

Other given methods are used to prepare colloids.

9. Oxidation is the process of loss of electrons and hydrogen or gain of oxygen. *e.g.*,

$$Na \longrightarrow Na^+ + e^-$$

10. Albumin is a globular protein as its molecules are folded with spheroidal shapes. Moreover, in it intramolecular H-bonding is found.

Note Fibrous proteins are held together by intermolecular H-bonding.

11. CrO_2Cl_2 is known as chromyl chloride, a compound formed as intermediate in chromyl chloride test of chloride ion.

12.
$$NH_3 + NaNO_2 \longrightarrow NaNH_2 + HNO_2$$
$$NH_4Cl + NaNO_2 \xrightarrow{\Delta} NaCl + N_2 + 2H_2O$$
$$N_2O + Cu \longrightarrow CuO + N_2$$
$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$$

Three out of the four processes give dinitrogen but $NH_4Cl + NaNO_2$ give the pure N_2.

13. Atomic number is the fundamental property of elements and Modern Periodic Table is based on it.

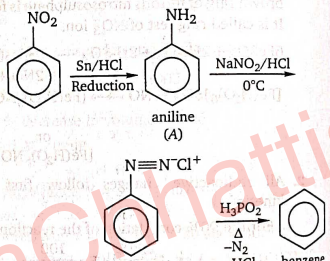
14. Let the strong dibasic acid is H_2A.

$$\underset{0.05\ M}{H_2A} \longrightarrow \underset{2 \times 0.05\ M}{2H^+} + A^{2-}$$

$$[H^+] = 2 \times 0.05 = 0.1\ M$$

$$\therefore \quad pH = -\log [H^+] = -\log (0.1)$$
$$= 1.0$$

15.



16. Let in $[Fe(CN)_6]^{4-}$ oxidation state of Fe is x.

$$x + (-1)6 = -4$$
$$x = -4 + 6 = +2$$
$$Fe(26) = [Ar]\ 3d^6 4s^2$$
$$Fe^{2+} = [Ar]\ 3d^6$$

$$\underbrace{\boxed{\uparrow\downarrow}\ \boxed{\uparrow}\ \boxed{\uparrow}\ \boxed{\uparrow}\ \boxed{\uparrow}}_{\text{four unpaired electrons}}$$

i.e., $n = 4$

We know that

magnetic moment, $\mu = \sqrt{n(n+2)}$
$$= \sqrt{4(4+2)}$$
$$= \sqrt{24}\ BM$$

17.
$$CH_3CHO \xrightarrow{[O]} \underset{\text{acetic acid}\ (A)}{CH_3COOH} \xrightarrow[\text{HVZ reaction}]{P,\ Cl_2} \underset{(B)}{ClCH_2COOH} \xrightarrow{KCN} \underset{(C)}{CNCH_2COOH} \xrightarrow{H^+,\ H_2O} HOOCCH_2COOH$$
or
$$CH_2(COOH)_2$$
malonic acid
(D)

18. Molarity $= \dfrac{\text{Mass} \times 1000}{\text{Molar mass} \times \text{volume}}$

$$= \frac{1000}{18}$$

($\because$ Molar mass of $H_2O = 18\ g\ mol^{-1}$)

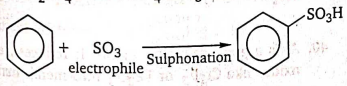
$$= 55.55\ M$$

19. Among the given potassium bisulphite is an inorganic preservative because it liberates SO_2 gas which is a strong insectiside.

$$KHSO_3 \xrightarrow{HCl} KCl + H_2O + SO_2 \uparrow$$

Note Benzoic acid and vinegar are also preservatives but they, being organic compound, are called organic preservatives.

20. $2H_2SO_4 \longrightarrow HSO_4^- + H_3O^+ + SO_3$



21. $\overset{5}{CH_3}—\overset{4}{CH}=\overset{3}{CH}—\overset{2}{C}\equiv\overset{1}{CH}$

[Root word-pent, for double bond-en suffix and for ≡ bond-yne suffix]

Thus, IUPAC name is pent-3-en-1-yne.

22. According to first law of thermodynamics,

Internal energy, $\Delta U = q + W$

(where, q = heat and W = workdone $= -p\Delta V$)

23. For the reaction,

$$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$$
$$E^\circ_{cell} = E^\circ_{Fe^{2+}/Fe} - E^\circ_{Zn^{2+}/Zn}$$
$$= -0.41 - (-0.76)$$
$$[\because Fe^{2+} + 2e^- \longrightarrow Fe;\ E^\circ = -0.41\ V$$
$$\text{and } Zn^{2+} + 2e^- \longrightarrow Zn;\ E^\circ = -0.76\ V]$$
$$= -0.41 + 0.76$$
$$= +0.35\ V$$

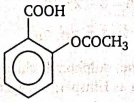
24. Aqueous solution of hypo when treated with Cl_2 gives $NaHSO_4$ as the product.

$$Na_2S_2O_3 + 4Cl_2 + 5H_2O \longrightarrow 2NaHSO_4 + 8HCl$$

25. Number of subshells $= n^2$

$$= (3)^2 = 9$$

26. 2-acetoxy benzoic acid is also known as aspirin which has analgesic and antipyretic properties.



2-acetoxy benzoic acid
or
aspirin

(**Note** Analgesic are pain releiver drugs and antipyretics bring down body temperature during high fever.)

27. Due to lanthanoid contraction the size of zirconium (Zr) and hafnium (Hf) are almost similar and hence, they are called chemical twins.

Note Zr and Hf have similar chemical properties.

28. The bases present in DNA are adenine (A), guanine (G), cytosine (C) and thymine (T). Uracil (U) is found in RNA, not in DNA.

29.
$$\underset{(C_2H_2)}{CH\equiv CH} \xrightarrow[H_2SO_4,\ H_2O]{Hg^{2+}} CH_2=CHOH \longleftrightarrow \underset{\text{acetaldehyde}\ (A)}{CH_3CHO}$$
$$\underset{(A)}{CH_3CHO} \xrightarrow{[O]} \underset{\text{acetic acid}\ (B)}{CH_3COOH}$$

30.
$$\overset{+7}{KMnO_4} \longrightarrow \overset{+4}{MnO_2}$$
$\therefore$ Number of electrons transferred $= 7 - 4$
$= 3$

$$\overset{+7}{KMnO_4} \longrightarrow Mn^{2+}$$
$\therefore$ Number of electrons transferred $= 7 - 2$
$= 5$

31. $2SO_2 + O_2 \rightleftharpoons 2SO_3 + \Delta H$

Since the reaction is exothermic (*i.e.*, heat is evolved), increase in temperature favours the backward reaction, *i.e.*, decomposition of SO_3 according to Le-chatelier's principle.

32. Crystalline solids are anisotropic *i.e.*, have different physical properties in different directions, hard and dense.

Note Isotropy is the property of amorphous solids.

33. O^{2-} ion $= 8 + 2 = 10e^-$

$= 1s^2, 2s^2, 2p^6$

34. Benzene sulphonyl chloride ($C_6H_5SO_2Cl$) is known as Hinsberg's reagent.

Note Hinsberg's reagent is used to distinguish 1°, 2° and 3° amines.

35. Sulphide ores are concentrated by froth floatation process.

Given mineral	Formula
Horn silver	AgCl
Haematite	Fe_2O_3
Bauxite	$Al_2O_3 \cdot 2H_2O$
Cinnabar	HgS

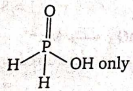
Thus, cinnabar being a sulphide ore is concentrated by froth floatation process.

36. Finding empirical formula

Element	Percentage (x)	Atomic mass (y)	Molar ratio (x/y)	Simplest molar ratio
C	40	12	$\frac{40}{12} = 3.33$	$\frac{3.33}{3.33} = 1$
H	13.33	1	$\frac{13.33}{1} = 13.33$	$\frac{13.33}{3.33} = 4$
N	46.67	14	$\frac{46.67}{14} = 3.33$	$\frac{3.33}{3.33} = 1$

Thus, the empirical formula of the compound is CH_4N.

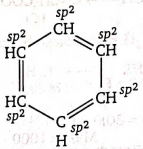
37. The structure of H_3PO_2 is as



∴ It is a monobasic acid, and for monobasic acids M = N.

∴ $0.2\,M\ H_3PO_2 = 0.2\,N\ H_3PO_2$

38. The structure of benzene is as



(∵ Double bonded C atoms are generally sp^2 hybridised).

39. Propyne because of the presence of acidic H-atom, reacts with ammoniacal $AgNO_3$ and gives grey/white silver propynilide. Propene, on the other hand, does not have any acidic H-atom, so it does not react with ammoniacal $AgNO_3$. Hence, these two are separated by this reagent.

$$CH_3C \equiv CH + AgNO_3 + NH_4OH \longrightarrow \underset{\text{silver propynilide}}{CH_3C \equiv CAg \downarrow} + NH_4NO_3 + H_2O$$

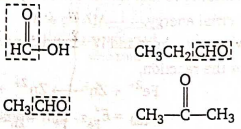
$$CH_3CH{=}CH_2 + AgNO_3 + NH_4OH \longrightarrow \text{No reaction}$$

40. Al is a good reducing agent. It reduces metal oxides like Cr_2O_3 or Fe_2O_3 into metals during metal extraction.

$$2Al + Fe_2O_3 \longrightarrow Al_2O_3 + 2Fe$$

$$2Al + Cr_2O_3 \longrightarrow Al_2O_3 + 2Cr$$

41. Silver mirror test is given by only the compounds containing —CHO group.



(No—CHO group)

Hence, CH_3COCH_3 does not give silver mirror test.

42. In Frenkel defect, the ions only change their place but not missing from the crystal lattice and hence, it does not affect the density of crystal or in other words density remains the same.

43. In flame test, sodium burns with golden gellow flame.

44. As the size of halogen atom increases, H—X (where, X = halogen) bond length increases, *i.e.*, H—X bond becomes weaker and it readily gives a H-atom, or readily brings about the reduction of other species.

Thus, HI is the strongest reducing agent because of weakest H—I bond.

45. Cetyltrimethyl ammonium bromide is a detergent, *i.e.*, it is a surfactant.

Note Soap and detergent are called surface active reagents or surfactants.

46. For the first order reaction,

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

For 20% completion of the reaction,

$$k = \frac{2.303}{200} \log \frac{100}{100-20}$$

$$= \frac{2.303}{200} \log 1.25$$

$$= \frac{2.303 \times 0.0969}{200}$$

For 80% completion of the reaction,

$$k = \frac{2.303}{t} \log \frac{100}{100-80} = \frac{2.303}{t} \log 5$$

$$= \frac{2.303 \times 0.699}{t}$$

On putting the value of k, we get

$$\frac{2.303 \times 0.0969}{200} = \frac{2.303 \times 0.699}{t}$$

$$\therefore \quad t = \frac{200 \times 0.699}{0.0969}$$

$$= 1442 \text{ min} \approx 1400 \text{ min}$$

47. Noble metals like Au and Ag are extracted by cyanide process.

48. $$\frac{dN}{dt} = -kN$$

$$1.2 \times 10^{11} = kx = \frac{6.023 \times 10^{23}}{241}$$

$$k = \frac{1.2 \times 10^{11} \times 241}{6.023 \times 10^{23}}$$

We know that $t_{\frac{1}{2}} = \frac{0.693}{k}$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{1.2 \times 10^{11} \times 241} \text{ s}$$

$$= \frac{0.693 \times 6.023 \times 10^{23}}{1.2 \times 10^{11} \times 241 \times 365 \times 24 \times 60 \times 60} \text{ yr}$$

$$= \frac{4173.939}{9.1} = 458 \text{ yr}$$

49. Half-life period, $t_{1/2} \propto \frac{1}{a^{n-1}}$

Given, $(t_{1/2}) \propto a$

On comparing

$$a \propto \frac{1}{a^{n-1}}$$

or $$a^1 \propto a^{1-n}$$

$$1 = 1 - n$$

$$1 - 1 = -n$$

$$n = 0$$

Hence, the order of the reaction is 0.

50. The heat of neutralisation of a strong acid (HCl) and a strong base (NaOH) is always –13.7 kcal or –57.5 kJ.

Mathematics

1. Given, $\mathbf{p} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$

and $\mathbf{q} = b\mathbf{i} + c\mathbf{j} + a\mathbf{k}$

Now,

$$\cos\theta = \frac{\mathbf{p}\cdot\mathbf{q}}{|\mathbf{p}||\mathbf{q}|} = \frac{ab+bc+ca}{\sqrt{a^2+b^2+c^2}\sqrt{a^2+b^2+c^2}}$$

We know that,

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$\Rightarrow \quad ab+bc+ca = \frac{(a+b+c)^2-(a^2+b^2+c^2)}{2}$$

$$\therefore \quad \cos\theta = \frac{(a+b+c)^2-(a^2+b^2+c^2)}{2(a^2+b^2+c^2)}$$

Maximum value of $\cos\theta$, when $a = b = c = k$

i.e., $$\cos\theta = \frac{(3k)^2-(3k^2)}{2(3k^2)} = \frac{6k^2}{6k^2} = 1$$

$$\Rightarrow \quad \theta = 0°$$

Minimum value of $\cos\theta$, when $a+b+c=0$

$$\Rightarrow \cos\theta = \frac{0-(a^2+b^2+c^2)}{2(a^2+b^2+c^2)} = \frac{-(a^2+b^2+c^2)}{2(a^2+b^2+c^2)}$$

$$= -\frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{2\pi}{3}$$

$$\therefore \quad \theta \in [0, 2\pi/3]$$

2. We know that,

$$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401$$

$$7^5 = 16807$$

For $\lambda = 1, \mu = 3, 7, 11, \ldots, 97$

$\therefore$ Favourable cases = 25

For $\lambda = 2, \mu = 4, 8, 12, \ldots, 100$

$\therefore$ Favourable cases = 25

Similarly for every λ, favourable $\mu = 25$

$\therefore$ Total favourable cases $= 100 \times 25$

$\therefore$ Required probability $= \dfrac{100\times 25}{100\times 100}$

$$= \frac{1}{4}$$

3. Given,

$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$ and taking common from C_1

$$\Rightarrow \quad (1+2x)\begin{vmatrix} 1 & x & x \\ 1 & 1 & x \\ 1 & x & 1 \end{vmatrix}$$

$$+ (3-x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \quad (1+2x)\begin{vmatrix} 1 & x & x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$$+ (3-x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x & 0 \\ 0 & 0 & -x \end{vmatrix} = 0$$

$$\Rightarrow \quad (1+2x)(1-x)^2 + (3-x)x^2 = 0$$

$$\Rightarrow \quad (1+2x)(1+x^2-2x) + (3-x)x^2 = 0$$

$$\Rightarrow \quad 1+2x+x^2+2x^3-2x-4x^2+3x^2$$

$$-x^3 = 0$$

$$\Rightarrow \quad x^3+1=0 \Rightarrow x^3 = -1$$

$$\therefore \quad x = -1, -\omega, -\omega^2$$

$$\therefore \quad x^{2005} + \frac{1}{x^{2005}} = x^{2004}\cdot x + \frac{1}{x^{2004}\cdot x}$$

$$= (x^3)^{668}\cdot x + \frac{1}{(x^3)^{668}\cdot x}$$

$$= x(-1)^{668} + \frac{1}{(-1)^{668}\cdot x}$$

$$= x + \frac{1}{x}\times\frac{x^2}{x^2}$$

$$= x + \frac{x^2}{-1} = x - x^2$$

When $x = -\omega$

$$x - x^2 = -\omega - (-\omega)^2$$

$$= -\omega - \omega^2 = 1$$

when $x = -\omega^2$

$$x - x^2 = -\omega^2 - (-\omega^2)^2$$

$$= -\omega^2 - \omega^4 = -\omega^2 - \omega$$

$$= 1$$

4. Given, $f(x) = (x-1)^{2/3}$

It is clear that, $f(x)$ is continuous everywhere.

On differentiating w.r.t. 'x,' we get

$$f'(x) = \frac{2}{3(x-1)^{1/3}}$$

Here, at $x = 1$, $f'(x)$ is not defined, so it is not differentiable at $x = 1$.

So, option (a), (b) and (c) are not correct.

Hence, option (d) is correct.

5. $2^x + 2^y = 2^{x+y}$

On differentiating w.r.t. 'x,' we get

$$2^x \log 2 + 2^y \log 2\frac{dy}{dx} = 2^{(x+y)}\log 2\left(1+\frac{dy}{dx}\right)$$

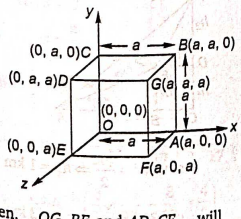
At $x = y = 1$,

$$\Rightarrow 2\log 2 + 2\log 2\frac{dy}{dx} = 2^2\log 2 + 2^2\log 2\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}(2\log 2) = -2\log 2$$

$$\therefore \quad \frac{dy}{dx} = -1$$

6. Let side of the cube be $= a$



Then, OG, BE and AD, CF will be four diagonals.

DR's of $OG = (a, a, a) = (1, 1, 1)$

DR's of $BE = (-a, -a, a) = (1, 1, -1)$

DR's of $AD = (-a, a, a) = (-1, 1, 1)$

DR's of $CF = (a, -a, a) = (1, -1, 1)$

Let DR's of line be l, m and n.

Therefore, angle between line and diagonal

$$\cos\alpha = \frac{l+m+n}{\sqrt{3}}, \cos\beta = \frac{l+m-n}{\sqrt{3}}$$

$$\cos\gamma = \frac{-l+m+n}{\sqrt{3}}$$

and $$\cos\delta = \frac{l-m+n}{\sqrt{3}}$$

$$\therefore \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$$

$$= \frac{1}{3}[(l+m+n)^2 + (l+m-n)^2 + (-l+m+n)^2$$

$$+ (l-m+n)^2]$$

$$= \frac{1}{3}[4(l^2+m^2+n^2)] = \frac{4}{3} \quad (\because l^2+m^2+n^2 = 1)$$

7. The Newton Raphson method converges fast if $f'(\alpha)$ is large.

8. $(1+i)^n + (1-i)^n$

Using polar form,

$$1+i = \sqrt{2}(\cos\pi/4 + i\sin\pi/4)$$

and $$1-i = \sqrt{2}(\cos\pi/4 - i\sin\pi/4)$$

Now, $$(1+i)^n + (1-i)^n$$

$$= [\sqrt{2}(\cos\pi/4 + i\sin\pi/4)]^n$$

$$+ [\sqrt{2}(\cos\pi/4 - i\sin\pi/4)]^n$$

$$= (\sqrt{2})^n\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right)$$

$$+ (\sqrt{2})^n\left(\cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right)$$

$$= 2(\sqrt{2})^n\cos\frac{n\pi}{4} = (\sqrt{2})^{n+2}\cos\frac{n\pi}{4}$$

9. Let $I = \int_0^1 \sin^{-1} x\, dx$

Put $x = \sin\theta$

$\Rightarrow dx = \cos\theta\, d\theta$

$$I = \int_0^{\pi/2} \theta \cos\theta \, d\theta$$

Using integration by parts, we get

$$I = [\theta \sin\theta]_0^{\pi/2} - \int_0^{\pi/2} 1 \times \sin\theta \, d\theta$$

$$= [\pi/2 - 0] - [-\cos\theta]_0^{\pi/2}$$

$$= \pi/2 - (1) = \pi/2 - 1$$

10. $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$

Using series of $\sin x$ and $\cos x$,

$$\Rightarrow \lim_{x \to 0} \frac{x\left[1 + a\left(1 - \frac{x^2}{2!} + \dots\right)\right] - b\left(x - \frac{x^3}{3!} + \dots\right)}{x^3} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{x(1+a) - \frac{x^3}{2!} + \dots - bx + b\frac{x^3}{3!} + \dots}{x^3} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{x(1 + a - b) + x^3\left(\frac{b}{6} - \frac{a}{2}\right) + \dots}{x^3} = 1$$

For existence of finite limit,

$$1 + a - b = 0 \quad \dots(i)$$

Then, $\lim_{x \to 0} \frac{x^3\left(\frac{b}{6} - \frac{a}{2}\right) + \dots}{x^3} = 1$

$$\Rightarrow \frac{b}{6} - \frac{a}{2} = 1$$

$$\Rightarrow b - 3a = 6 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$a = \frac{-5}{2} \text{ and } b = -\frac{3}{2}$$

11. $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{y + x\tan\frac{y}{x}}{x}$$

It is a homogeneous differential equation.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{vx + x\tan\frac{vx}{x}}{x}$$

$$\Rightarrow x\frac{dv}{dx} = v + \tan v - v$$

$$\Rightarrow x\frac{dv}{dx} = \tan v$$

$$\Rightarrow \frac{dv}{\tan v} = \frac{dx}{x}$$

$$\Rightarrow \cot v \, dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\log\sin v = \log x + \log C$$

$$\Rightarrow \log\sin v = \log xC$$

$$\Rightarrow \sin v = xC$$

$$\therefore \sin\frac{y}{x} = Cx \quad \left(\because \text{Put } v = \frac{y}{x}\right)$$

12. $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP.

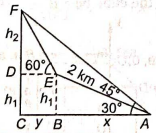
$$\therefore (a^{x/2})^2 = \log_x a \times \log_b x$$

$$a^x = \frac{\log_b a}{\log_b x} \times \log_b x$$

$$a^x = \log_b a$$

$$\Rightarrow x = \log_a(\log_b a)$$

13. Let A be the foot of the mountain and F be the summit of the mountain. In right angled ΔABE,



$$\sin 30° = \frac{h_1}{2} \Rightarrow h_1 = 1 \text{ km}$$

and $\cos 30° = \frac{x}{2} \Rightarrow x = \sqrt{3}$ km

In right angled ΔDEF,

$$\tan 60° = \frac{h_2}{y}$$

$$\Rightarrow y = \frac{h_2}{\sqrt{3}}$$

And in right angled ΔACF,

$$\tan 45° = \frac{h_1 + h_2}{x + y}$$

$$\Rightarrow x + y = h_1 + h_2$$

$$\sqrt{3} + \frac{h_2}{\sqrt{3}} = 1 + h_2$$

$$\Rightarrow h_2 = \sqrt{3}$$

$$\sqrt{3} - 1 = \frac{(\sqrt{3} - 1)h_2}{\sqrt{3}}$$

$\therefore$ Height of mountain $= h_1 + h_2$

$$= 1 + \sqrt{3}$$

14. $\mathbf{a} \cdot \mathbf{p} + \mathbf{b} \cdot \mathbf{q} + \mathbf{c} \cdot \mathbf{r}$

$$= \frac{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]} + \frac{\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]} + \frac{\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$$

$$= \frac{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]} + \frac{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]} + \frac{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]}$$

$$= 1 + 1 + 1$$

$$= 3$$

15. DR's of given lines are 2, 5, 4 and 1, 2, –3.

$$\therefore \cos\theta = \frac{2 \times 1 + 5 \times 2 + 4 \times (-3)}{\sqrt{2^2 + 5^2 + 4^2}\sqrt{1^2 + 2^2 + (-3)^2}}$$

$$= \frac{2 + 10 - 12}{\sqrt{45}\sqrt{14}}$$

$$\cos\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

16. $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$

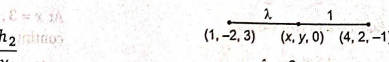
$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1 - \left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1 - \left(\frac{3}{5}\right)^2}\right]$$

$$= \sin^{-1}\left(\frac{3}{5} \times \frac{15}{17} - \frac{8}{17} \times \frac{4}{5}\right)$$

$$= \sin^{-1}\left(\frac{45}{85} + \frac{32}{85}\right) = \sin^{-1}\left(\frac{77}{85}\right)$$

$$= \cos^{-1}\frac{36}{85}$$

17. At XOY plane, Z-coordinate is zero *i.e.*, $(x, y, 0)$.



$$\Rightarrow \frac{-\lambda + 3}{\lambda + 1} = 0 \Rightarrow \lambda = 3$$

$\therefore$ Ratio = 3 : 1

18. The arithmetic calculations in a computer are performed by ALU.

19. $x\frac{dy}{dx} - y = 2x^2$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$$

On compare with linear differential equation

$$\frac{dy}{dx} + Py = Q$$

$$\Rightarrow P = -\frac{1}{x}$$

$$\therefore \text{IF} = e^{\int P\,dx} = e^{\int -\frac{1}{x}dx} = e^{-\log x}$$

$$= e^{\log_e \cdot \frac{1}{x}} = \frac{1}{x}$$

20. Since, $\frac{1 + 4p}{4}, \frac{1 - p}{4}, \frac{1 - 2p}{2}$ are probabilities of mutually exclusive events, then

$$0 \le \frac{1 + 4p}{4} + \frac{1 - p}{4} + \frac{1 - 2p}{2} \le 1$$

$$\Rightarrow 0 \le \frac{1 + 4p + 1 - p + 2 - 4p}{4} \le 1$$

$$\Rightarrow 0 \le 4 - p \le 4$$

$$\Rightarrow -4 \le -p \le 0$$

$$\Rightarrow 0 \le p \le 4 \quad \dots(i)$$

Also, $0 \le \frac{1 + 4p}{4} \le 1, 0 \le \frac{1 - p}{4} \le 1$

and $0 \le \frac{1 - 2p}{2} \le 1$

$$\Rightarrow 0 \le 1 + 4p \le 4, 0 \le 1 - p \le 4$$

and $\quad 0 \le 1-2p \le 2$

$\Rightarrow \quad -\frac{1}{4} \le p \le \frac{3}{4}, -3 \le p \le 1$

and $\quad -\frac{1}{2} \le p \le \frac{1}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$$0 \le p \le \frac{1}{2}$$

21. Since, $\frac{a+b}{1-ab}$, b and $\frac{b+c}{1-bc}$ are in AP.

$\therefore \quad b - \frac{a+b}{1-ab} = \frac{b+c}{1-bc} - b$

$\Rightarrow \quad \frac{-a(b^2+1)}{1-ab} = \frac{c(b^2+1)}{1-bc}$

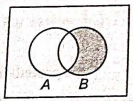
$\Rightarrow \quad -\left(\frac{1-ab}{a}\right) = \frac{1-bc}{c}$

$\Rightarrow \quad -\frac{1}{a} + b = \frac{1}{c} - b$

$\Rightarrow \quad 2b = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \quad a, \frac{1}{b}$ and c are in HP.

22.



$\bar{A} \cap B$ = Common region

$= B - A \cap B$

$\therefore \quad P(\bar{A} \cap B) = P(B) - P(A \cap B)$

23. $m = {}^nC_2 = \frac{n(n-1)}{2}$

Then, ${}^mC_2 = {}^{\frac{n^2-n}{2}}C_2$

$$= \frac{\left(\frac{n^2-n}{2}\right)\left(\frac{n^2-n}{2}-1\right)}{2!}$$

$$= \frac{n(n-1)(n^2-n-2)}{8} = \frac{n(n-1)(n+1)(n-2)}{8}$$

$$= \frac{(n+1)(n)(n-1)(n-2)}{8}$$

$$= 3 \times {}^{n+1}C_4$$

24. $f(x) = a - (x-3)^{8/9} \Rightarrow f'(x) = -\frac{8}{9(x-3)^{8/3}}$

At $x = 3$, $f'(x)$ does not exist but $f(x)$ is continuous.

$\Rightarrow$ Hence, no value of x exist.

25. Given, $n = 4$

Here, $\quad h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

$\therefore$ It divide into 4 subintervals.

x	0	0.25	0.5	0.75	1
Ordinate	y_0	y_1	y_2	y_3	y_4
$y = \frac{1}{1+x^2}$	1	$\frac{1}{1.0625} = 0.94117$	$\frac{1}{1.25} = 0.8$	$\frac{1}{1.5625} = 0.64$	$\frac{1}{2} = 0.5$

$\therefore$ By simpson rule,

$$\int_0^1 f(x)\,dx = \frac{h}{3}[(y_0 + y_4) + 2y_2 + 4(y_1 + y_3)]$$

$$= \frac{1/4}{3}[(1+0.5) + 2\times 0.8 + 4(0.94117 + 0.64)]$$

$$= \frac{1}{12}(1.5 + 1.6 + 6.32468)$$

Given, $\quad = \frac{9.42468}{12} = 0.78539$

26. Given, $\quad f(x) = \log x$

$\Rightarrow \quad f(\log x) = \log(\log x)$

On differentiating w.r.t. x, we get

$\therefore \quad f'(\log x) = \frac{1}{\log x} \times \frac{1}{x} = (x \log x)^{-1}$

27. The control structure of IF-THEN is a multiple selection because IF-THEN use a Nested loop.

28. Given, $\quad b + c = 3a$

$$\therefore \cot\frac{B}{2}\cot\frac{C}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

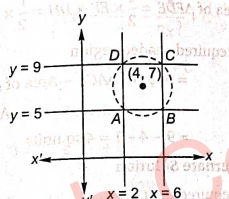
$$= \frac{s}{s-a} = \frac{(a+b+c)/2}{\frac{a+b+c}{2} - a}$$

$$= \frac{(a+b+c)/2}{(b+c-a)/2}$$

$$= \frac{(a+3a)/2}{(3a-a)/2} = \frac{4a}{2a} \quad (\because b + c = 3a)$$

$$= 2$$

29. $x^2 - 8x + 12 = 0$



$\Rightarrow \quad (x-6)(x-2) = 0$

$\Rightarrow \quad x = 2, 6$

and $\quad y^2 - 14y + 45 = 0$

$\Rightarrow \quad (y-9)(y-5) = 0$

$\Rightarrow \quad y = 5, 9$

Firstly, we draw the lines $x = 2$, $x = 6$ and $y = 5$, $y = 9$.

The intersection of these lines formed a square $ABCD$.

A circle is circumscribed on the square $ABCD$ whose centre is (4, 7).

30. $\tan 57° = \tan(45° + 12°)$

$\Rightarrow \quad \tan 57° = \frac{\tan 45° + \tan 12°}{1 - \tan 45° \tan 12°}$

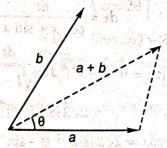
$\Rightarrow \quad \tan 57° = \frac{1 + \tan 12°}{1 - \tan 12°}$

$\Rightarrow \quad \tan 57° - \tan 57° \tan 12° = 1 + \tan 12°$

$\therefore$

$\tan 57° - \tan 12° - \tan 57° \tan 12° = 1 = \tan 45°$

31. $\tan\theta = \frac{2\sin 60°}{2 + 2\cos 60°}$



$$\tan\theta = \frac{2 \times \frac{\sqrt{3}}{2}}{2 + 2 \times \frac{1}{2}}$$

$$= \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$\therefore \quad \theta = 30°$

32. Let $\quad I = \int_0^{\pi/2}(\sqrt{\tan x} + \sqrt{\cot x})\,dx$

$$\left[\because \text{if } \int_0^{2a} f(x) = \int_0^{2a} f(2a-x), \text{ then } \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx\right]$$

$$\therefore \quad I = 2\int_0^{\pi/4}(\sqrt{\tan x} + \sqrt{\cot x})\,dx$$

$$= 2\int_0^{\pi/4}\left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right)dx$$

$$= 2\int_0^{\pi/4}\frac{(\sin x + \cos x)}{\sqrt{\sin x \cos x}}\,dx$$

$$= 2\sqrt{2}\int_0^{\pi/4}\frac{\sin x + \cos x}{\sqrt{2\sin x \cos x}}\,dx$$

(on multiplying numerator and denominator by $\sqrt{2}$)

$$= 2\sqrt{2}\int_0^{\pi/4}\frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}}\,dx$$

$$\left[\because (\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2\sin x \cos x \quad \therefore \quad 2\sin x \cos x = 1 - (\sin x - \cos x)^2\right]$$

Put $\sin x - \cos x = t \Rightarrow \cos x + \sin x = \frac{dt}{dx}$

$\Rightarrow \quad dx = \frac{dt}{\cos x + \sin x}$

$\therefore \quad I = 2\sqrt{2}\int_{-1}^{0} \frac{dt}{\sqrt{1-t^2}}$

$= 2\sqrt{2}[\sin^{-1} t]_{-1}^{0}$

$= 2\sqrt{2}[\sin^{-1} 0 - \sin^{-1}(-1)]$

$= 2\sqrt{2}[0 + \sin^{-1}(1)]$

$= 2\sqrt{2}\left(\frac{\pi}{2}\right) = \pi\sqrt{2}$

33. Given integral is $\int_0^1 x\,dx$.

Suppose, we divided this into four equal intervals.

i	0	1	2	3	4
x_i	0	0.25	0.5	0.75	1
y_i	0	0.25	0.5	0.75	1

$\therefore$ By Trapezoidal rule,

$\int_0^1 dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + y_3) + y_4]$

$= \frac{1-0}{2\times 4}[0 + 2(0.25 + 0.5 + 0.75) + 1]$

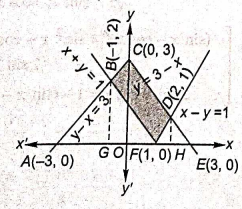
$= \frac{1}{8}[2(1.5) + 1]$

$= \frac{1}{8}(3+1) = \frac{1}{2}$

34. Given curves are $y = |x-1|$ and $y = 3 - |x|$.

The intersection point of lines $x + y = 1$ and $y - x = 3$ is $B(-1, 2)$.

and intersection point of lines $x - y = 1$ and $y = 3 - x$ is $D(2, 1)$.



Now, Area of

$\Delta ACE = \frac{1}{2} \times AE \times OC = \frac{1}{2} \times 6 \times 3 = 9$

Area of $\Delta ABF = \frac{1}{2} \times AF \times BG = \frac{1}{2} \times 4 \times 2 = 4$

and

Area of $\Delta FDE = \frac{1}{2} \times EF \times DH = \frac{1}{2} \times 2 \times 1 = 1$

$\therefore$ Required shaded region

= Area of ΔACE − Area of ΔABF − Area of ΔFDE

$= 9 - 4 - 1 = 4$ sq units

Alternate Solution

$\therefore$ Required area

$= \int_{-1}^{3} [(3 - |x|) - (|x-1|)]\,dx$

$= \int_{-1}^{0} [3 + x - (1-x)]\,dx + \int_0^1 [(3-x) - (1-x)]\,dx + \int_1^2 [(3-x) - (x-1)]\,dx$

$= \int_{-1}^{0} (2+2x)\,dx + \int_0^1 (2)\,dx + \int_1^2 (4-2x)\,dx$

$= [2x + x^2]_{-1}^{0} + 2[x]_0^1 + [4x - x^2]_1^2$

$= [0 - (-2+1)] + 2(1-0) + [8 - 4 - (4-1)]$

$= 1 + 2 + 1 = 4$ sq units

35. Total cases $= 6 \times 6 = 36$

Favourable cases $= 1$ *i.e.*, $(1, 1)$

$\therefore$ Required probability $= \frac{1}{36}$

36. $\frac{\text{Projection of } \mathbf{a} \text{ on } \mathbf{b}}{\text{Projection of } \mathbf{b} \text{ on } \mathbf{a}} = \frac{\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|}}{\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}}$

$= \frac{|\mathbf{a}|}{|\mathbf{b}|} = \frac{\sqrt{2^2 + (-3)^2 + (6)^2}}{\sqrt{4+4+1}}$

$= \frac{\sqrt{49}}{\sqrt{9}} = \frac{7}{3}$

37. $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$

$\Rightarrow \quad df(x) = \left(4x^3 - \frac{3}{x^4}\right)dx$

On integrating both sides, we get

$f(x) = \frac{4x^4}{4} + \frac{3}{3x^3} + C$

$\Rightarrow \quad f(x) = x^4 + \frac{1}{x^3} + C$

At $\quad f(2) = 0,$

$\Rightarrow \quad 0 = 2^4 + \frac{1}{2^3} + C$

$C = -16 - \frac{1}{8} = -\frac{129}{8}$

$\therefore \quad f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$

38. At $x = 2$,

RHL $= \lim_{h \to 0} [2 + h] + [-(2+h)]$

$= \lim_{h \to 0} [2 + (-2-h)] = 2 - 3 = -1$

Since, $f(x)$ is continuous at $x = 2$

$\therefore \quad$ RHL $= f(2)$

$\therefore \quad \lambda = -1$

39. $A^2 - A + I = 0$

On multiplying A^{-1} both sides, we get

$A^{-1}(A)A - A^{-1}A + A^{-1}I = 0$

$IA - I + A^{-1} = 0 \quad (\because I = A^{-1}A)$

$\Rightarrow \quad A^{-1} = I - A$

40. Let $\quad y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$

and $\quad z = \sqrt{1 - x^2}$

Put $\quad x = \cos\theta$

$\because \quad y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$

$= \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$= \sec^{-1}(\sec 2\theta) = 2\theta$

and $\quad z = \sqrt{1 - \cos^2\theta} = \sin\theta$

On differentiating w.r.t. θ, we get

$\Rightarrow \quad \frac{dy}{d\theta} = 2$ and $\frac{dz}{d\theta} = \cos\theta$

$\therefore \quad \frac{dy}{dz} = \frac{dy/d\theta}{dz/d\theta} = \frac{2}{\cos\theta} = \frac{2}{x}$

$\Rightarrow \quad \left.\frac{dy}{dz}\right|_{\left(x = \frac{1}{2}\right)} = \frac{2}{1/2} = 4$

41. $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$

Order = 3 and degree = 2

42. $x = a\cos^3\theta,\ y = a\sin^3\theta$

On differentiating w.r.t. θ, we get

$\Rightarrow \quad \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta,\ \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

$\Rightarrow \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$

$= -\tan\theta$

$\therefore \sqrt{1 + (dy/dx)^2} = \sqrt{1 + (-\tan\theta^2)}$

$= |\sec\theta|$

43. Now,

$(1100110011)_2 = 1\times 2^9 + 1\times 2^8 + 0\times 2^7 + 0\times 2^6 + 1\times 2^5 + 1\times 2^4 + 0\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0$

$= 512 + 256 + 0 + 0 + 32 + 16 + 0 + 0 + 2 + 1$

$= 819$

and $(1101001011)_2 = 1\times 2^9 + 1\times 2^8 + 0\times 2^7 + 1\times 2^6 + 0\times 2^5 + 0\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0$

$$= 512 + 256 + 0 + 64 + 0 + 0 + 8 + 0 + 2 + 1$$

$$= 843$$

$$\therefore \quad (1101001011)_2 - (1100110011)_2$$

$$= 843 - 819 = 24$$

$$= (11000)_2$$

2	24	0
2	12	0
2	6	0
2	3	1
	1	

44. $2 \cdot 6 \cdot 10 \ldots (4n-6)(4n-2)$

$$= 2^n\{1 \cdot 3 \cdot 5 \ldots (2n-3)(2n-1)\}$$

$$= \frac{2^n\{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots \cdot (2n-3)(2n-2)(2n-1)2n\}}{2 \cdot 4 \cdot 6 \ldots (2n-2)(2n)}$$

$$= \frac{2^n \times (2n)!}{2^n(1 \cdot 2 \cdot 3 \cdot \ldots \cdot n)}$$

$$= \frac{(2n)!}{n!} = {}^{2n}P_n$$

45. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow \quad e_1 = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{a^2 + b^2}{a^2}}$$

and $\quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

$$\Rightarrow \quad \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$\Rightarrow \quad e_2 = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{\frac{a^2 + b^2}{b^2}}$$

$$\therefore \quad \frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2}$$

$$= 1$$

46. Given, differential equation can be written as

$$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$$

or $\quad \frac{dx}{dy} + \frac{1}{1 + y^2} \cdot x = \frac{\tan^{-1} y}{1 + y^2}$

It is a linear differential equation of the form

$$\frac{dx}{dy} + Px = Q$$

where, $\quad P = \frac{1}{1 + y^2}$

and $\quad Q = \frac{\tan^{-1} y}{1 + y^2}$

$\therefore$ Integrating factor $= e^{\int P\, dy}$

$$= e^{\int \frac{1}{1 + y^2} dy}$$

$$= e^{\tan^{-1} y}$$

$\therefore$ Solution is

$$e^{\tan^{-1} y} \cdot x = \int \frac{e^{\tan^{-1} y} \tan^{-1} y}{1 + y^2} dy$$

Put $\tan^{-1} y = t$ in right hand side,

$$\Rightarrow \quad \frac{1}{1 + y^2} dy = dt$$

$$\therefore \quad x e^{\tan^{-1} y} = \int \underset{\text{II}}{e^t} \underset{\text{I}}{t}\, dt$$

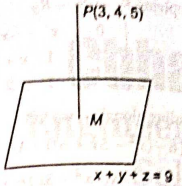
$$= te^t - \int 1 \cdot e^t\, dt + C$$

$$= te^t - e^t + C$$

$$= \tan^{-1} y \cdot e^{\tan^{-1} y} - e^{\tan^{-1} y} + C$$

$$\Rightarrow \quad xe^{\tan^{-1} y} = (\tan^{-1} y - 1)\, e^{\tan^{-1} y} + C$$

47. Let M be the foot of perpendicular from $P(3, 4, 5)$ to the given plane, then PM is normal to the plane. So, its DR's are $(1, 1, 1)$.



$\therefore$ Equation of line PM is

$$\frac{x - 3}{1} = \frac{y - 4}{1} = \frac{z - 5}{1} = k \text{ (say)}$$

$$\Rightarrow \quad x = k + 3,\ y = k + 4,\ z = k + 5$$

Let coordinate of M be

$$(k + 3, k + 4, k + 5) \quad \ldots(i)$$

Since, point M lies on a plane $x + y + z = 9$.

$\therefore$ It satisfies the equation of plane.

$$\therefore \quad 1 \cdot (k + 3) + 1 \cdot (k + 4) + 1 \cdot (k + 5) = 9$$

$$\Rightarrow \quad 3k + 12 = 9$$

$$\Rightarrow \quad 3k = -3$$

$$\Rightarrow \quad k = -1$$

Put $k = -1$ in Eq. (i), we get

The coordinate of $M(-1 + 3, -1 + 4, -1 + 5)$

i.e., $\quad M(2, 3, 4)$

48. Let point of contact be (x_1, y_1). Given, equation of circle is

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

$\therefore$ Equation of chord of contact at point (x_1, y_1) is

$$xx_1 + yy_1 - 2(x + x_1) - 3(y + y_1) - 12 = 0$$

$$\Rightarrow \quad x(x_1 - 2) + y(y_1 - 3) - 2x_1 - 3y_1 - 12 = 0$$

But it is given that point of contact is $3x + 4y + 7 = 0$.

$$\therefore \quad \frac{x_1 - 2}{3} = \frac{y_1 - 3}{4} = -\frac{(2x_1 + 3y_1 + 12)}{7}$$

On taking Ist and last terms,

$$7x_1 - 14 = -(6x_1 + 9y_1 + 36)$$

$$\Rightarrow \quad 13x_1 + 9y_1 + 22 = 0 \quad \ldots(i)$$

On taking IInd and last terms,

$$7y_1 - 21 = -(8x_1 + 12y_1 + 48)$$

$$\Rightarrow \quad 8x_1 + 19y_1 + 27 = 0 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x_1 = -1,\ y = -1$$

Hence, required point of contact is $(-1, -1)$.

49. Given equation is

$$x^2 - 7xy + 12y^2 = 0$$

On comparing with $ax^2 + 2hxy + by^2 = 0$

$$\therefore \quad a = 1,\ h = -\frac{7}{2},\ b = 12$$

$$\therefore \quad m_1 + m_2 = -\frac{2h}{b}$$

and $\quad m_1 m_2 = \frac{a}{b}$

$$\Rightarrow \quad m_1 + m_2 = \frac{7}{12}$$

and $\quad m_1 m_2 = \frac{1}{12} \quad \ldots(i)$

Now, $\quad (m_1 - m_2) = \sqrt{(m_1 + m_2)^2 - 4m_1 m_2}$

$$= \sqrt{\left(\frac{7}{12}\right)^2 - 4 \times \frac{1}{12}}$$

$$= \sqrt{\frac{49}{144} - \frac{4}{12}}$$

$$= \sqrt{\frac{49 - 48}{144}}$$

$$= \sqrt{\frac{1}{144}} = \frac{1}{12}$$

$$\Rightarrow \quad m_1 - m_2 = \frac{1}{12} \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$m_1 = \frac{1}{3}$$

and $\quad m_2 = \frac{3}{12}$

Now, $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2} = \dfrac{\frac{1}{3} - \frac{3}{12}}{1 + \frac{1}{3} \times \frac{3}{12}}$

$$\tan\theta = \frac{\frac{1}{12}}{\frac{13}{12}} = \frac{1}{13} \neq 0$$

Hence, given equation is a non-perpendicular straight lines.

50. Since, given matrix is a singular.

$$\therefore \quad \begin{vmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{vmatrix} = 0$$

On applying $R_1 \to R_1 - R_2$

$$\begin{vmatrix} 0 & x & -x \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{vmatrix} = 0$$

Expanding along R_1, we get

$$-x[6 - (x^2 + 5x + 6)] - x(-3 + x + 3) = 0$$

$$\Rightarrow \quad -x(-x^2 - 5x) - x(x) = 0$$

$$\Rightarrow \quad x(x^2 + 5x - x) = 0$$

$$\Rightarrow \quad x^2(x + 4) = 0$$

$$\Rightarrow \quad x = 0, -4$$

But it is given that, $x \in [-4, -1]$.

$$\therefore \quad 0 \notin [-4, -1]$$

Hence, only one solution exist.

Chhattisgarh PET
Engineering Entrance Exam
Solved Paper 2011

Physics

1. If in expression $X = 3YZ^2$, the dimensions of X and Z are same as the dimensions of capacity and magnetic field respectively, the dimensional formula of Y is

(a) $[M^{-3}L^{-2}T^{-4}A^{-1}]$ (b) $[ML^{-2}T^4]$
(c) $[M^{-3}L^{-2}T^4A^4]$ (d) $[M^{-3}L^{-2}T^8A^4]$

2. If L, C, R are respectively the inductance, capacitance and resistance, the quantities of dimensions same as of frequency are

(a) $\dfrac{1}{\sqrt{LC}}, \dfrac{R}{L}$ and $\dfrac{1}{RC}$
(b) $\sqrt{LC}, \dfrac{L}{R}$ and RC
(c) $\sqrt{\dfrac{L}{C}}$, LR and $\dfrac{C}{R}$
(d) $\sqrt{\dfrac{C}{L}}, \dfrac{1}{LR}$ and $\dfrac{R}{C}$

3. The SI unit of thermal capacity is

(a) Joule
(b) Joule/kilogramme
(c) Joule/kelvin
(d) Joule/kelvin kilogramme

4. An object travels in a straight line one-third of the total distance with velocity v_1, second one-third distance with velocity v_2 and the rest one-third distance with velocity v_3. The average velocity of object will be

(a) $\dfrac{v_1 + v_2 + v_3}{3}$ (b) $\dfrac{v_1v_2 + v_2v_3 + v_3v_1}{3v_1v_2v_3}$
(c) $\sqrt{v_1v_2v_3}$ (d) $\dfrac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

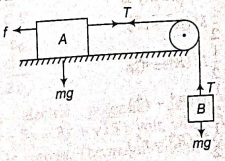
5. A ball is projected at an angle θ upwards from horizontal. The true statement is

(a) at each point of flight vertical component of momentum remains constant
(b) at each point of flight horizontal component of momentum remains constant
(c) at the highest point of flight, potential energy is minimum
(d) at the highest point of flight, kinetic energy is zero

6. A packet of mass m is dropped from an aeroplane moving at a height h above the ground with a horizontal velocity u. If g is the acceleration due to gravity, the kinetic energy possessed by the packet on reaching the ground will be

(a) mgh (b) $\frac{1}{2}mu^2 + mgh$
(c) $\frac{1}{2}mu^2 - mgh$ (d) $mgh - \frac{1}{2}mu^2$

7. In the diagram below, a block A of mass 10 kg rests on a horizontal table. A massless string attached with it passes over a frictionless pulley attached at the end of table with another block B at its free end. If coefficient of friction between the block A and table surface is 0.2, the minimum mass of block B needed to start motion in block A is



(a) 2 kg (b) 0.2 kg (c) 5 kg (d) 10 kg